Stability and Control of Mobile Communications Systems With Time Varying Channels

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Abstract

Consider the forward link of a mobile communications system with a single transmitter and rather arbitrary randomly time varying channels connecting the base to the mobiles. Data arrives at the base in some random way (and might have a bursty character) and is queued according to the destination until transmitted. The main issues are the allocation of transmitter power and time to the various queues in a queue- and channel-state dependent way to assure stability and good operation. The control decisions are made at the beginning of the (small) scheduling intervals. Stability methods are used to allocate time and power. Many schemes of current interest can be handled: For example, CDMA with control over the bit interval and power per bit, TDMA with control over the time allocated, power per bit, and bit interval, as well as arbitrary combinations. There might be random errors in transmission which require retransmission. The channel-state process might be known or only partially known. The details of the scheme are not directly involved; all essential factors are incorporated into a "rate" and "error" function. The system and channel process are scaled by speed. Under a stability assumption on a model obtained from the "mean drift," and some other natural conditions, it is shown that the scaled physical system can be controlled to be stable, uniformly in the speed, for fast enough speeds. Owing to the non-Markov nature of the problem, we use the perturbed Liapunov function method, which is very useful for the analysis of non-Markovian systems. Finally, the stability method is used to actually choose the power and time allocations. The allocation will depend on the Liapunov function. But each such function corresponds loosely to an optimization problem for some performance criterion. Since there is a choice of Liapunov functions, various performance criteria can be taken into account in the allocations. The resulting controls are quite reasonable. The power of the method is due to the rather general conditions under which it works and the reasonableness of the controls.

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1 Introduction

We consider the problem of power and time control for the forward link of a mobile communications system when the connecting channels are time varying. There are K queues at the base station, each receiving *data* according to some random process, which allows burstiness. Time is divided into small scheduling intervals of length $\bar{\Delta}^{s,n}$ and the transmitter decisions concerning power allocation, etc., are made at the beginning of the intervals. With the appropriate use of pilot signals or other estimation methods, it is becoming increasingly practical to estimate key properties of the channel and to use this information to help allocate transmitter power and time among the competing queues. One could suppose, for example, that some receivers make measurements of their own connecting channels via a pilot signal, and then pass information on the channel state back to the base. This information determines the set of acceptable power levels and bit intervals. This approach could greatly improve the performance [1]. The channel state is denoted by j, which is *vector-valued* and indicates the states of *all* of the K individual channels. It is assumed that the channel state takes only a finite number of values. There is no natural impediment to the use of a continuous state space, but it requires more detail. The individual components of the channel state might or might not be mutually independent. The actual physical situation that corresponds to a channel state j is unimportant. Each state j corresponds to an allowed set of usage patterns. Given ones assessment of the channel (the channel state), one need only know the probability of bit error at the receiver under an arbitrary power allocation and the allowed rates of transmission. These are assumed to be known functions of the channel state. For simplicity, it is assumed that the channel state does not change during a scheduling interval. The arrivals to and contents of the queues and rates of transmission are measured in terms of packets. The data "arrival rates" and the rate of change of the channel state are "high." The actual transmission schemes that are allowed are quite general. They can be based on TDMA, CDMA, bit interval control, or on various combinations. The channel state space need not be finite, provided that the true states are divided into a finite number of aggregated groups that are indexed by j, and that decisions are made on the basis of the aggregated group or its estimate.

Generally, to best accommodate issues of fairness and the particular performance criteria of interest, this power and time allocation should depend on the lengths of the queues as well as on the current knowledge of the (vector) state of the channel. There might be either complete or only partial knowledge of the channel. Furthermore, depending on the noise levels and the character of the individual channel, a received packet might be found to contain too many errors, and will need to be retransmitted. The resulting optimal control problem is quite difficult. The dimension (K) might be high, the set of queue length process might not be Markovian under any control scheme, and uncertainty of the channel state can lead to a complicated filtering problem.

The difficulties of direct solution, the high speed of the system and the high rate of change of the channel state process suggest an approach to the control

problem that is based on an "averaged" system. It will be seen that such an approach can be carried out under quite general conditions for a great variety of systems, based on TDMA, CDMA, or various combinations.

The approach is as follows. We consider a sequence of systems, scaled by n. The rates of service and arrivals are $O(n)$. There are $\nu \in (0,1)$ and a process $L(\cdot)$ such that the the channel-state process can be represented as $L^n(\cdot)$, where $L^n(t) = L(n^\nu t)$. More will be said about $L(\cdot)$ later. In fact, the representation of the channel state process in the form $L^n(t) = L(n^\nu t)$ is just a convenience for the notation. It is not necessary, provided that $L^n(\cdot)$ changes "fast enough." In any particular application, the value of n is fixed. But the results show that for high enough system speed and fast enough channel variations, we will have stability under the obtained (very reasonable) controls. Define $\bar{x}_i^n(t)$ to be $1/n$ times the number of packets in queue i at time t. Define $\bar{x}^n(t) = (\bar{x}_i^n(t), i \le K)$.

In typical current stability results for queueing networks, one uses assumptions that allow the current queue to be represented in terms of a Markov process [2]. For example, the interarrival intervals and service requirements are independent and those for a particular class are identically distributed. Then one represents the network in terms of the "residual" arrival and service times and the current content. Fluid approximations are derived for $Q(Mt)/M$ as $M \to \infty$, where $Q(t)$ is the queue size at time t. Then, under appropriate additional conditions, one shows that stability of the fluid approximation implies various stability properties of the queue. Such a method does not work for our case. First, the assumptions do not necessarily allow a Markov representation. Second, we are concerned with a family of queues, indexed by both the speed n and the controls, and are interested in getting stabilizing controls that have other useful properties as well. Finally, even if there is a fluid approximation in the above "scaled and time squeezing sense" for each n, it in itself won't lead to the "uniformity" results that are desired. Nevertheless, there is a "mean drift," and the stability results for the queues depend on a stability assumption concerning the system whose dynamics are this mean drift. Because of this, and to motivate this equation for the randomly time varying channel, in Section 3 a fluid approximation to $\bar{x}^n(\cdot)$ is obtained under assumptions on the data arrival processes that are stronger than those used in the stability analysis in Sections 4–6. This approximation depends on the actual power and time allocation rule that is used as well as on the (channel state and power allocation dependent) probabilities of acceptance of a received packed. As noted above (A4.1), with a little extra work the various arrival and service "rates" $(\bar{\lambda}_i^a, \bar{\lambda}_i^d(j))$ could be made queue-state dependent.

Definition of stability, uniformly in large n. Suppose that there are $q_0 < \infty$ and a real-valued function $f(\cdot)$ such that the following holds: For σ_0^n *any* random time for $\bar{x}^n(\cdot)$, and $\sigma_1^n = \min\{t \ge \sigma_0^n : |\bar{x}^n(t)| \le q_0\}$, we have

$$E_{\sigma_0^n}^n \left[\sigma_1^n - \sigma_0^n\right] \le f(\bar{x}^n(\sigma_0^n)) I_{\{|\bar{x}^n(\sigma_0^n)| > q_0\}}$$

with probability one for all large n and all such σ_0^n. [Here E_t^n denotes the expectation conditioned on *all* of the systems data to time t (arrival and service

times, actual channel states, estimated channel states, rejected packets).] Then we say that there is *stability, uniformly for large n.*

Remark. Although the queue state is normalized by n, the stability is for *each* individual n that is large. The qualifier "large" appears essentially since small scheduling intervals are required. Under certain conditions, the qualifier "large" can be dropped. See the comments at the end of Section 5. The assumptions (A4.1), (A5.2) and (A5.8) on the arrival processes allow bursts of order n. This makes sense since the usage should depend on the system bandwidth. Because of this, the buffers must be scaled accordingly and it makes sense to normalize by n.

The stability is proved using the perturbed Liapunov function methods of [8]. With this method, one starts with a basic Liapunov function that works for the fluid approximation. Then one finds a suitable perturbation that works for the actual physical problem.

Two canonical classes of physical models (and various combinations and extensions) are discussed in Section 2. These include the standard CDMA and TDMA. For expositional convenience the development is based on these models. It is seen that the approach and results are the same when the channel is only partly known or retransmission of poorly received packets is called for. A similar analysis can be used when there are multiple antennas [5] and frequencies. Multiple antennas and frequencies are used in the space-time coded OFDM (orthogonal frequency division multiplexing) approach [11, 14]. Part of the purpose of which is to provide 'space-time "diversity," to partially neutralize the effects of channel variations. But, via the approach of this paper and given sufficient information on the channel, one might find it preferable to use only the most appropriate combinations of antennas and frequencies for the various sources at any time.

There is a close connection between stability and optimal control. Considering the Liapunov function as a total cost function, the "conditional mean rate of change" of the Liapunov function is a cost rate, and the Liapunov function is an optimal cost function for the cost rate which is minimal over the possible control functions [7]. Thus, by varying the Liapunov function in a systematic manner, one gets solutions to a sequence of optimization problems. One can then exploit this flexibility to select a control that not only assures stability, but is approximately optimal with respect to some desired criterion. Some additional comments appear in Section 6.

Canonical models are given in Section 2. Section 4 gives and discusses some of the assumptions on the channel and arrival process that will be needed for the stability analysis. Assumptions on the Liapunov function and the stability proofs are in Section 5. An example is given in Section 6, where there is also a comparison of the obtained controls with optimal controls. It will be seen that the approach is quite natural and general for the problems of concern and yields results that are reasonable. We have chosen particular classes of packet arrival processes. But the general forms in which the associated conditions are

given are quite flexible. For problems of the type of Example 2.1, [1, 12] obtain rules for power allocation whose form is similar to ours and which are based on stability considerations, although the method uses large deviations estimates and the setup is Markovian. The reference [15] was perhaps the first to consider the problem of dynamic power allocation when the channels are time varying.

2 Classes of Models

In order to illustrate the numerous possible systems to which the methods are applicable, it is convenient to consider two basic classes. These will allow us to focus on the key points. It will be seen that, although the models in the two classes are quite different from a physical point of view, they are treated in essentially the same way, with similar limit forms, stability criteria, and controls. They can be combined, and this will be commented on later. Until further notice, suppose that the channel state is known, whether via the use of pilot signals or otherwise. The case of partially known channel state is discussed below (3.16) and in Sections 5 and 6.

In the first class of models, spelled out in Example 2.1 below, data from the selected queues are transmitted simultaneously, as in CDMA [16]. In addition, the bit interval might be controlled. In each scheduling interval the total available power is split among the queues in a queue- and channel-state-dependent way. In the second class of models, whose basic form is spelled out in Example 2.2 below, transmission is from only one queue at a time (e.g, as in TDMA) and both the power and the time allocated to the queues during a scheduling interval are controlled. Extensions to combined time and bit interval control are then noted. These power and/or time allocations depend on the information on the channel and queue state that is available at the transmitter at the start of the scheduling interval. The power allocation to each queue is constant during the interval. It will be seen that the types of controls in these two examples can be used simultaneously, and that the type can vary with the channel and queue state. Part of the scheduling interval might be taken by estimation of the channel state. But that is covered in our framework, since the results are based on what happens in an entire scheduling interval.

For notational convenience, we will work in *discrete time*. Assume that there is $\delta_n > 0$ such that all completed cell arrivals and transmissions (at the base station) occur only at integral multiples of δ_n. We simply suppose that $\delta_n = \alpha_0/n$ for some small α_0. The use of δ_n is just a bookkeeping device. To fix ideas, suppose that detection at the receiver is bit by bit, and ignore interuser interference (as is common with CDMA- and TDMA-type systems). Let the background noise in detection at the mobiles be white, with constant spectral density σ_i^2 for mobile i. Packets received at the mobile might be rejected (and need to be retransmitted) if the receiver decides that the decoded message contains too many errors. Interuser interference can be partly accounted for in determining the sets of allowable power combinations and the acceptance functions $q_i(\cdot)$ in (A3.3) or (A3.5). Unless otherwise specified, when dealing

with an arbitrary scheduling interval we will use the canonical variables j for the channel state and $\bar{x}$ for the queue state at the start of the interval, whatever it is.

Example 2.1. Power allocation, completely known channel state. In this scheme, in each scheduling interval one chooses a set of queues and transmits simultaneously from them, and power only is to be allocated. There is a total amount of power $n\bar{u}$ which is to be divided among the queues. Let $nu_i(j,\bar{x})$ denote the power allocated to queue i during the current scheduling interval. Thus,

$$\sum_i u_i(j,\bar{x}) \leq \bar{u}, \qquad \text{each channel state } j. \tag{2.1}$$

For each channel state j, the rate of (bit or packet) transmission (per unit time in the scheduling interval) is assumed to be proportional to the power allocated. This can be realized in several ways. One is to allow the possibility of several "spreading sequences" per queue [5]. An alternative is to allow the bit interval to be controllable (sometimes called "time diversity"). In particular, suppose that there are (perhaps queue-dependent) basic time intervals $\bar{\Delta}_i^{b,n}$, of which the actual bit interval is one of the multiples $m\bar{\Delta}_i^{b,n}, m = 1,\ldots,m_i$, for some given $m_i < \infty$. Thus, cutting the bit interval in half would require twice the power for the same signal to noise ratio. Set $m_i = 1$ if bit interval control is not allowed for queue i. Thus there are constants $\bar{\lambda}_i^d(j)$ such that the number of packets per unit time that can be sent from queue i under channel state j is $n\bar{\lambda}_i^d(j)u_i(j,\bar{x})$ for the allowed values of $u_i(j,\bar{x})$.[1] Additional details for one case are in the comments following the example. The chosen bit interval (duration) is constant during a scheduling interval.

With this setup, there is a basic packet interval $\bar{\Delta}_i^{p,n}$ (the number of bits per packet times $\bar{\Delta}_i^{b,n}$) and the packets can have the durations $m\bar{\Delta}_i^{p,n}, m = 1,\ldots,m_i$. The scheduling interval is assumed to be much larger than the packet interval in that $\bar{\Delta}^{s,n} >> m_i\bar{\Delta}_i^{p,n}$, but $n^\nu\bar{\Delta}^{s,n} \to 0$, where we recall that n^ν is the scaling for the channel state process. Thus the channel state changes slowly relative to the length of the scheduling interval. The allocated power has two roles. It determines the transmission rate as well as the probability that the receiver will accept a transmitted packet. The following remarks provide more detail on one case of interest.

Elaboration of the discussion of Example 2.1: A special case. Suppose that the actual signal power at the receiver output is (a real number) $\gamma_i(j)u_i(j,\bar{x})$. Then $\gamma_i(j)$ defines the channel state. Suppose that if the power allocation to any queue is not zero then it must be such that the signal to noise ratio per bit at the receiver is at least some given positive number (which might be channel- and queue-state dependent). Since detection is done bit by bit, it

[1] More generally, one could replace $n\bar{\lambda}_i^d(j)u_i(j,\bar{x})$ by $nf_i(j,u_i(j,\bar{x}))$, for some monotonic and continuous $f_i(\cdot)$. This is always a designers choice. For example, one might consider a nonlinear $f_i(j,\cdot)$ to account for delay spread or non white noise.

is the signal to noise power ratio for each bit interval that is of interest, since it determines the error distribution for the received packet. Hence it determines whether the received packet is accepted or rejected.

Suppose that if the channel state is j and data is transmitted from queue i, then we require a minimum required S/N ratio $c_i(j)$. Let $\bar{x}$ denote the state at the start of the current scheduling interval. Assume perfect synchronization (at the bit level) at the receiver, and denote the bit interval which is to be actually used by $\Delta = O(1/n)$. Then the probability of error per bit at receiver i is

$$P\left\{w(\Delta) \geq \sqrt{\gamma_i(j) n u_i(j,\bar{x})}\Delta\right\}$$

where $w(\Delta)$ is a random variable that is normally distributed with mean zero and variance $\sigma_i^2\Delta$. This probability equals

$$\frac{1}{\sqrt{2\pi}} \int_{[\gamma_i(j) n u_i(j,\bar{x})\Delta/\sigma_i^2(j)]^{1/2}}^{\infty} e^{-\xi^2/2} d\xi.$$

The signal/noise ratio of interest is

$$\frac{S}{N} = \frac{n u_i(j,\bar{x})\gamma_i(j)\Delta}{\sigma_i^2(j)} \geq c_i(j).$$

Thus, either don't transmit or choose $u_i(j,\bar{x})$ to yield a bit interval

$$\Delta = m\bar{\Delta}_i^{b,n} \geq \frac{c_i(j)\sigma_i^2(j)}{n\gamma_i(j)u_i(j,\bar{x})}$$

for some $m \leq m_i$. Let a_1 denote the inverse of the number of bits per packet. Then the rate of transmission $\bar{\lambda}_i^d(j)$ of packets per unit power for queue i is a_1/Δ. Equivalently, $\bar{\lambda}_i^d(j)$ is defined by

$$a_1 \frac{n\gamma_i(j)u_i(j,\bar{x})}{c_i(j)\sigma_i^2(j)} = n\bar{\lambda}_i^d(j)u_i(j,\bar{x}).$$

The scaled power is either zero or takes one of the values

$$u_i(j,\bar{x}) = \frac{c_i(j)\sigma_i^2(j)}{n\gamma_i(j)m\bar{\Delta}_i^{b,n}}, \quad m \leq m_i.$$

With this "variable bit length" model, the rate of transmission will always be proportional to power. The $m = 1$ lower bound puts an upper bound on the power that can be allocated to the queue.

We can allow a received packet to be rejected by the receiver if it decides that there are too many errors. Then the acceptance probability will also depend on the power and we let $q_i(j, u_i(j,\bar{x}))$ denote the probability that the received packet is accepted. [See A3.3 below.]

Example 2.2, which includes TDMA. In Example 2.1, it was supposed that the *total* power is constrained to $n\bar{u}$ at each time and that the transmissions

from all selected queues in a scheduling interval was done simultaneously. Alternatively, suppose that we schedule by dividing time as well as power in the following form of TDMA. Partition the μth scheduling interval into K subintervals (some subintervals might not be connected and some might have length zero) of total lengths $\Delta^{s,n}_{i,\mu}, i \leq K$, where $\sum_i \Delta^{s,n}_{i,\mu} = \bar{\Delta}^{s,n}$. The total length of the ith subinterval will be zero if we do not serve queue i in that scheduling interval. The ith subinterval will be connected if, once service on queue i starts in that scheduling interval, then it continues until all the allocated time is used. For services such as processor sharing, one cycles among the members of the subset of selected queues, and the subintervals will not be connected. Summarizing, in the μth scheduling interval, we transmit from queue i on a total time of length $\Delta^{s,n}_{i,\mu}$, which can depend on $i, \bar{x}$ and j.

Suppose that the bit interval does not depend on $\bar{x}$ but that it can depend on i, j. In the simplest scheme the power is constant at value $n\bar{u}$ at all times. However, since the scheduling intervals are short, if desired we can allow the power to vary within the interval, but to average to $n\bar{u}$ over the interval. In particular, let $nu_i(j, \bar{x})$ denote the power that is applied to queue i during the part of the scheduling interval that queue i is being worked on. [If $u_i(j, \bar{x}) \equiv \bar{u}$, then we are back to the simplest scheme.] Then, in lieu of the pointwise constraint (2.1), we now constrain *average total power* over the scheduling interval and (2.1) is replaced by

$$\sum_i u_i(j, \bar{x}) \frac{\Delta^{s,n}_{i,\mu}}{\bar{\Delta}^{s,n}} = \bar{u}, \quad \text{all } j. \tag{2.2}$$

Let $\bar{\lambda}^d_i(j)$ denote $1/n$ times the number of packets transmitted per unit of allocated time for queue i when the channel state is j. The effective rate of transmitting packets per unit time over the current scheduling interval for queue i is

$$\frac{n\bar{\lambda}^d_i(j)\Delta^{s,n}_{i,\mu}}{\bar{\Delta}^{s,n}}. \tag{2.3}$$

The presence of the factor $\bar{\lambda}^d_i(j)$ gives us the freedom to use bit intervals that depend on i, j. One chooses $u_i(j, \bar{x})$ to compensate for the channel dependent attenuation and receiver noise. In general, the length of the subintervals will depend on the state $(j, \bar{x})$ at the start of the scheduling interval, and we define the control over the time allocation to be $v_i(j, \bar{x}) = \Delta^{s,n}_{i,\mu}/\bar{\Delta}^{s,n}$.

Extensions to Example 2.2. The setup of Example 2.2 can be extended in many ways. A more complicated form keeps the time division, but allows the bit intervals to depend on the queue state, as in Example 2.1. *Redefine* $\bar{\lambda}^d_i(j)$ to be the *rate per unit of allocated time per unit power.* Then, with $q_i(j, u_i(j, \bar{x}))$ denoting the acceptance probability, the effective rate of transmitting packets per unit time for the current scheduling interval can be written as

$$\frac{n\bar{\lambda}^d_i(j)u_i(j, \bar{x})q_i(j, u_i(j, \bar{x}))\Delta^{s,n}_{i,\mu}}{\bar{\Delta}^{s,n}}. \tag{2.4}$$

This is the form (but with partially unknown channel state as well) that will be used in the sequel when extensions to Example 2.2 are referred to.

Further complicating the model, we can allow the subintervals to overlap, which means that transmission is allowed from several queues simultaneously, perhaps a combination of CDMA and TDMA. For example, for part of the interval we might transmit from one subset of queues via CDMA and similarly from another subset on the rest of the interval.

3 A Fluid Approximation

Fluid approximations will be obtained for special cases of the models of the types of Examples 2.1 and 2.2 and their extensions. It will be seen that the conditions, results, and proofs, are essentially the same for all cases. In fact Theorem 3.3 for the extension of Example 2.2 covers all cases. The essential requirements are a mixing condition on the channel state process, the continuity of the control functions, and a weak convergence assumption on the scaled arrival process. We start with Example 2.1 where the channel state is known. A convenient representation of the transmission term is given and then the main theorem is stated and proved. Theorem 3.2 concerns the case where the channel state is only partially known and the proof requires only minor adjustments. Theorem 3.3 deals with the extension to Example 2.2 and, again, the proof requires only minor alterations.

The fluid limits are given to show that they arise naturally even for the non-Markov setup and are similar for all cases, and also because essentially the same channel process is used for the stability analysis. The stability proofs do not depend on the fact that there is a fluid approximation, only on the fact that the differential equations (3.8), (3.17), or (3.22) (according to the case) are stable under the controls which are used. Under the conditions that are used for the stability analysis in Sections 4 and 5 (which allow more general arrival processes), there might not be a fluid limit. Under the conditions of this section, the ODE's do represent the fluid approximations, but they also represent the "average drift terms" under the conditions that are used for the stability analysis. [2]

3.1 Example 2.1.

Definitions and assumptions. Consider the model of Example 2.1 where the assigned power to queue i is $nu_i(j, \bar{x})$ at each time in the current scheduling interval, and (2.1) holds. Let $\bar{A}_i^n(t)$ be $1/n$ times the number of packets that have arrived at queue i by time t. Let $\bar{D}_i^n(t)$ denote $1/n$ times the number of packets whose transmission from queue i was completed *and* which were

[2] With a little extra work, the rates $\bar{\lambda}_i^a$ and $\bar{\lambda}_i^d(j)$ could depend (continuously) on the queue-state.

accepted by the receiver by time t. Then

$$\bar{x}_i^n(t) = \bar{x}_i^n(0) + \bar{A}_i^n(t) - \bar{D}_i^n(t). \tag{3.1}$$

The arrivals are assumed to occur in batches, with $v_{i,l}^{a,n}$ denoting the size of the ith batch. Let $I_{i,l\delta_n}^{\alpha,n}$, $\alpha = a$ (resp., $\alpha = d$) denote the indicator function of an arrival (resp., completed transmission) at queue i at time $l\delta_n$, and let $I_{i,l\delta_n}^{r,n}$ denote the indicator function that such a transmitted packet was accepted by the receiver and does not need to be retransmitted. Then

$$\bar{D}_i^n(t) = \frac{1}{n} \sum_{l=0}^{t/\delta_n} I_{i,l\delta_n}^{d,n} I_{i,l\delta_n}^{r,n}. \tag{3.2}$$

When t/δ_n appears as an index of summation, as in (3.2), take the *integer* part. If there is an arrival of a batch at time $k\delta_n$, denote the size by $v_{i,k\delta_n}^{a,n}$. The scaled arrival process is

$$\bar{A}_i^n(t) = \frac{1}{n} \sum_{l=0}^{t/\delta_n} v_{i,l\delta_n}^{a,n} I_{i,l\delta_n}^{a,n}, \tag{3.3}$$

Assumption 3.3 formalizes the white noise and detection assumptions made in the Introduction. Let E_t^n denote the expectation conditioned on all of the system data to time t. In all weak convergence statements, the Skorohod topology [3] is used on the space of functions on the value space which are right continuous and have left hand limits.

A3.1. $\{\bar{x}^n(0)\}$ *is tight.*

A3.2. *There are constants* $\bar{\lambda}_i^a$ *and* $\bar{v}_i^a$ *such that the process* $\bar{A}_i^n(\cdot)$ *converges weakly to the nonrandom process with values* $t\bar{\lambda}_i^a\bar{v}_i^a$.

A3.3. *The probability that the* l*th packet sent from queue* i *will be rejected, conditioned on the data up to the time that the transmission of the packet is complete, is* $q_i(j, u_i(j, \bar{x}))$, *which is continuous in* u_i *for each* i, j, *where* $(j, \bar{x})$ *is the (channel, queue) state at the start of the scheduling interval in which the packet is sent. Also,* $q_i(j, 0) = 0$.

A3.4. *There are* $\Pi(j)$ *such that for each* i, j, *and* $T < \infty$, *the integral*

$$\int_t^T E_t^n \left[I_{\{L^n(s)=j\}} - \Pi(j) \right] ds = n^{-\nu} \int_{n^\nu t}^{n^\nu T} E_t^n \left[I_{\{L(s)=j\}} - \Pi(j) \right] ds$$

is well-defined, bounded uniformly in $\omega, n, t \le T$, *where* ω *is the canonical variable of the probability space, and it converges to zero uniformly in* $t \le T$ *and in* ω, *as* $n \to \infty$,

Remarks on the assumptions. (A3.2) simply says that for high enough system speed and arrival "rates," $1/n$ times the number of packets arriving in

any time interval is approximately the mean packet size times the mean arrival rate times the interval. The form used in the stability analysis is more general, allowing non-degenerating "Poisson jumps" and other forms of burstiness of the scaled process. Condition (A3.4) is not restrictive. Commonly, there is enough "mixing" in $L(\cdot)$ so that the right hand integral is bounded uniformly in (t,T). For example, if $L(\cdot)$ is an ergodic finite-state Markov chain, then the convergence of the conditional expectation is exponentially fast, whatever the initial condition, and the right-hand integral is bounded.

Representations for the transmission term. Let $\bar{x}_i^n[s]$ and $L^n[s]$ denote the values of the queue and channel states at the *start* of the scheduling interval containing time s. For simplicity in the representation of the transmission term, suppose that a queue i with $\bar{\lambda}_i^d(j) > 0$ can be served with a particular power allocation in the scheduling interval only if the number of packets that it contains at the beginning of the interval is at least what could be served during that interval under the power allocation, unless this condition is impossible to meet for all queues. I.e., for any channel state j there is no idle time unless all queues with positive $\bar{\lambda}_i^d(j)$ have very small content. This assumption will have no effect on the stability analysis, which concerns queue-state values that are not small. More precisely, the scaled number of packets that can be transmitted from queue i in the μth scheduling interval when the channel state is j and under a given power allocation is proportional to both the scheduling interval and the power and is

$$d_i^n(j) = \frac{1}{n}\left[n\bar{\lambda}_i^d(j)u_i(j,\bar{x}^n(\mu\bar{\Delta}^{s,n}))\bar{\Delta}^{s,n}\right] \to 0, \text{ as } n \to \infty.$$

Thus in Theorems 3.1–3.3 we suppose that (when the channel state is j) if $u_i(j,\bar{x}^n[l\delta_n]) > 0$, then $\bar{x}_i^n[l\delta_n] \geq d_i^n(j)$. Actually, the the requirement in the theorems that $u_i(j,\cdot)$ be continuous and have zero value at $\bar{x} = 0$, covers the above restriction, since it assures that negligible power is applied to queues with negligible content.

Rewrite (3.2) by centering the summands about their "conditional mean values:"

$$\bar{D}_i^n(t) = \left[\bar{D}_i^n(t) - \bar{D}_i^{0,n}(t)\right] + \bar{D}_i^{0,n}(t), \tag{3.4}$$

where the centering term is

$$\bar{D}_i^{0,n}(t) = \delta_n \sum_{l=0}^{t/\delta_n} \sum_j \bar{\lambda}_i^d(j)u_i(j,\bar{x}^n[l\delta_n])q_i(j,u_i(j,\bar{x}^n[l\delta_n]))I_{\{L^n[l\delta_n]=j\}}I_{\{\bar{x}_i^n[l\delta_n]>d_i^n(j)\}}. \tag{3.5}$$

The right hand indicator function is redundant since it will be unity if and only if $u_i(j,\bar{x}^n[l\delta_n]) > 0$, and it will be dropped. The centered term (the bracketed

term in (3.4)) is thus

$$\frac{1}{n}\sum_{l=0}^{t/\delta_n}\left[I^{d,n}_{i,l\delta_n}I^{r,n}_{i,l\delta_n}-n\delta_n\sum_j\bar\lambda^d_i(j)u_i(j,\bar x^n[l\delta_n])q_i(j,u_i(j,\bar x^n[l\delta_n]))I_{\{L^n[l\delta_n]=j\}}\right]. \tag{3.6}$$

Define the occupation measure process $T^n(j,t)=\int_0^t I_{\{L^n(s)=j\}}ds$. As will be seen in Theorem 3.1, the limit $T(j,\cdot)$ satisfies

$$T(j,t)=\Pi(j)t. \tag{3.7}$$

This and the assumed continuity of the $u_i(j,\cdot)$ will yield the limit form

$$\begin{aligned}\bar x_i(t) &= \bar x_i(0)+\int_0^t b_i(\bar x(s),u(\bar x(s))ds,\\ b_i(\bar x,u(\bar x)) &= \bar\lambda^a_i\bar v^a_i-\sum_j\bar\lambda^d_i(j)u_i(j,\bar x)q_i(j,u_i(j,\bar x))\Pi(j).\end{aligned} \tag{3.8}$$

In Theorems 3.1–3.3, we suppose that $u_i(j,0)=0$ in order to simplify the treatment on the boundary and concentrate the power and time allocations on the queues with non-negligible content. With this scheme, all of the power might not be used if $\bar x$ is very small, even if not zero. But the assumptions are used to get a canonical fluid model only, and the stability analysis in Sections 4–6 concern large $\bar x$ values.

Theorem 3.1. *Assume* (A3.1)–(A3.4), *let the* $u_i(j,\cdot)$ *be continuous with* $u_i(j,0)$ $=0$, *and suppose that the channel state is known. Assume* (3.1) *and the model of Example* 2.1. *Then* $\{\bar x^n(\cdot)\}$ *is tight. The limit of any weakly convergent subsequence satisfies* (3.8).

Remark on the continuity of the controls. Continuity is assumed to facilitate the proof. The result using relaxed controls which is given after the theorem always holds, and implies that (3.8) still holds with the understanding that the possible values of the control at any point in the set of discontinuity are in the convex hull of the values in the "neighboring" points.

Proof. To evaluate $\bar D^n_i(\cdot)$, start by considering the centered sum, for each j,

$$\frac{1}{n}\sum_{l=0}^{t/\delta_n}I^{d,n}_{i,l\delta_n}\left[I^{r,n}_{i,l\delta_n}-q_i(j,u_i(j,\bar x^n[l\delta_n]))\right]I_{\{L^n[l\delta_n]=j\}}. \tag{3.9}$$

Under (A3.3) the summands are martingale differences. The variance of (3.9) is bounded by

$$\frac{C}{n^2}E\sum_{l=1}^{t/\delta_n}I^{d,n}_{i,l\delta_n}I_{\{L^n[l\delta_n]=j\}},$$

where C is an upper bound on the expectation of $\left[I^{r,n}_{i,l\delta_n} - q_i(j, u_i(j, \bar{x}^n[l\delta_n]))\right]^2$, conditioned on the channel state j, $\bar{x}^n[l\delta_n]$, and the fact that the transmission of a packet is completed at $l\delta_n$. Since the sum is $O(t/n)$, the processes defined by (3.9) converge weakly to the "zero" process.

Next consider the sum (which is the centering in (3.9))

$$\sum_j \frac{1}{n} \sum_{l=0}^{t/\delta_n} I^{d,n}_{i,l\delta_n} q_i(j, u_i(j, \bar{x}^n[l\delta_n])) I_{\{L^n[l\delta_n]=j\}}. \tag{3.10}$$

Note that the coefficient of $I^{d,n}_{i,l\delta_n}$ in (3.10) for $l\delta_n$ in the μth scheduling interval depends on the queue and channel states at the start of the interval only, and not otherwise on l or μ. Fix the channel state at the start of a scheduling interval to be j, and take the sum in (3.10) only over those l that correspond to that scheduling interval. Then, recalling the definitions of $\bar{\lambda}^d_i(j)$ as $1/n$ times the rate per unit power, and of $[l\delta_n]$ as the time at the start of the scheduling interval containing $l\delta_n$, the contribution to the sum (3.10) of these indices l is just (for any $l\delta_n$ in the interval and any channel state j)

$$\bar{\lambda}^d_i(j) u_i(j, \bar{x}^n[l\delta_n]) q_i(j, u_i(j, \bar{x}^n[l\delta_n])) I_{\{L^n[l\delta_n]=j\}} \bar{\Delta}^{s,n}.$$

Thus, by summing over j and modulo an asymptotically negligible error, (3.10) equals

$$\delta_n \sum_{l=0}^{t/\delta_n} \sum_j \bar{\lambda}^d_i(j) u_i(j, \bar{x}^n[l\delta_n])) q_i(j, u_i(j, \bar{x}^n[l\delta_n])) I_{\{L^n[l\delta_n]=j\}}, \tag{3.11}$$

which is (3.5). Thus (3.9) equals (3.6).

The centering process (3.11) is tight; in fact, it is asymptotically Lipschitz continuous on the infinite time interval. Hence by the above approximations $\{\bar{x}^n(\cdot)\}$ is tight and is asymptotically Lipschitz continuous on any finite interval. Take a weakly convergent subsequence and, for notational simplicity, abuse notation and index it by n. The weak convergence, continuity of the limit, and the fact that $\bar{\Delta}^{s,n} \to 0$, implies that for any $T < \infty$, and in the sense of probability,

$$\lim_n \sup_{t \le T} |x^n(t) - x^n[t]| = 0. \tag{3.12}$$

We need only characterize the limit of the integral approximation to (3.11), namely of,

$$\sum_j \int_0^t \bar{\lambda}^d_i(j) u_i(j, \bar{x}^n[s]) q_i(j, u_i(j, \bar{x}^n[s])) T^n(j; ds). \tag{3.13}$$

where we write $I_{\{L^n[s]=j\}} ds = T^n(j; ds)$. By (3.12), one can replace $x^n[s]$ by $x^n(s)$ without changing the limits.

Proof that $dT(j;t)/dt = \Pi(j)$. A perturbed test function method [8] will be used. It is sufficient to work on an arbitrary time interval $[0,T]$. Let $\mathcal{F}_t^n$ denote the minimal σ-algebra which measures all of the arrival, service, acceptance and channel state processes to time t. Let E_t^n denote the associated conditional expectation. For any countable sequence $\delta \to 0$, let $f^{\delta,n}(\cdot)$ be a sequence of $\mathcal{F}_t^n$-adapted real-valued processes. Fix n. Following [8, 6], we say that p–$\lim_{\delta\to 0} f^{\delta,n}(\cdot) = 0$ if $\sup_{\delta,t} E|f^{\delta,n}(t)| < \infty$ and $\lim_{\delta\to 0} E\left|f^{\delta,n}(t)\right| = 0$ for each t. Define the operator $\hat{A}^n$ as follows. We say that $f^n(\cdot) \in \mathcal{D}(\hat{A}^n)$ (the domain of $\hat{A}^n$) and $\hat{A}^n f^n(t) = g^n(t)$ if

$$p\text{–}\lim_{\delta\to 0}\left[\frac{E_t^n f^n(t+\delta) - f^n(t)}{\delta} - g^n(t)\right] = 0.$$

For $s,t \geq 0$, we have $T^n(j;t+s) - T^n(j;t) \leq s$. Thus $\{T^n(j,\cdot)\}$ is tight in the Skorohod topology and all limits are Lipschitz continuous. Let $f(\cdot)$ be a bounded real-valued test function on $[0,\infty)$, with compact support and whose derivatives up to second order are continuous and let $f_v(\cdot)$ denote the first derivative. Then $\hat{A}^n f(T^n(j:t)) = f_v(T^n(j;t)) I_{\{L^n(t)=j\}}$. For $t \leq T$, define the test function perturbation

$$\delta f^n(t) = f_v(T^n(j;t)) \int_t^T E_t^n \left[I_{\{L^n(s)=j\}} - \Pi(j)\right] ds,$$

and set $f^n(t) = f(T^n(j;t)) + \delta f^n(t)$. By (A3.4), $\sup_t |\delta f^n(t)| \to 0$, uniformly in ω as $n \to \infty$. Note that, for $t \leq T$,

$$\hat{A}^n \delta f^n(t) = \\ -f_v(T^n(j;t))\left[I_{\{L^n(t)=j\}} - \Pi(j)\right] + O(1)\left|\int_t^T E_t^n \left[I_{\{L^n(s)=j\}} - \Pi(j)\right] ds\right|. \tag{3.14}$$

By (A3.4), the last term on the right of (3.14) also goes to zero uniformly in t,ω, as $n \to \infty$. Thus, by (A3.4) and (3.14),

$$\lim_{n\to\infty} p\text{–}\lim_{\delta\to 0}\left|\frac{E_t^n f^n(t+\delta) - f^n(t)}{\delta} - f_v(T^n(j;t))\Pi(j)\right| = 0.$$

It follows from this [8, Theorem 1, Chapter 4], the tightness of $\{T^n(j,\cdot)\}$, and the fact that the perturbations $\delta f^n(t)$ converge to zero uniformly in t,ω, that $T^n(j,\cdot)$ converges weakly to the process with values $\Pi(j)t$.

Now, use the approximation

$$\int_0^t \sum_j \bar{\lambda}_i^d(j) u_i(j,\bar{x}^n(s)) q_i(j, u_i(j,\bar{x}^n(s))) dT^n(j;s)$$

of (3.13), the weak convergence, and the continuity of $q_i(j,\cdot)$ and $u_i(j,\cdot)$, to get the last term of (3.8). ■

A representation of (3.5) in terms of relaxed controls. Relaxed controls [9] will not be needed in the sequel, but they do allow us to get a limit without any continuity restrictions on the controls $u_i(j,\cdot)$. We comment briefly on the changes in the theorem. Define $\bar{u}_i^n(j,t) = u_i(j, \bar{x}^n[t])$. Define the control space of K-tuples $U = \{\alpha = (\alpha_1, \ldots, \alpha_K) : \sum_i \alpha_i \leq \bar{u}\}$. Define the relaxed control representation $r_i^n(j : \cdot)$ of $\bar{u}_i^n(j,\cdot)$ via its derivative with respect to Lebesgue measure $r_{i,t}^n(j : \cdot)$; namely, define

$$r_{i,t}^n(j : d\alpha_i) = I_{\{\bar{u}_i^n(j,t) \in d\alpha_i\}} I_{\{L^n(t)=j\}},$$

and

$$r_i^n(j : A \times [0,t]) = \int_0^t \int_A r_{i,s}^n(j, d\alpha_i) ds,$$

for any Borel set $A \in [0, \bar{u}]$. Note that

$$\sum_i r_i^n(j; [0,\bar{u}] \times t) = T^n(j : t).$$

Write (3.5) as (modulo an asymptotically negligible error, due to the approximation of the sum by an integral)

$$\int_0^t \sum_j \int_{[0,\bar{u}]} \bar{\lambda}_i^d(j) \alpha_i q_i(j, \alpha_i) r_{i,s}^n(j : d\alpha_i) ds. \tag{3.15}$$

With the weak topology used on the space of relaxed controls, the sequence of relaxed controls $\{r_i^n(j,\cdot)\}$ is always tight for each i, j [9]. Thus, taking a weakly convergent subsequence for all i, j, yields the limit form

$$\bar{x}_i(t) = \bar{x}_i(0) + \bar{\lambda}_i^a \bar{v}_i^a t - \int_0^t \sum_j \int_{[0,\bar{u}]} \bar{\lambda}_i^d(j) \alpha_i q_i(j, \alpha_i) r_{i,s}(j : d\alpha_i) ds. \tag{3.16}$$

Incompletely known channel state. In Theorem 3.1, it was supposed that the channel state process $L^n(t)$ is known at the beginning of each scheduling interval. This might be a good approximation. But, whatever the means of estimation, it will never be perfect and any practical method must be robust with respect to estimation errors. We will consider a reasonable formulation for which the required changes are minor. Let $L^{e,n}(\cdot)$ denote the estimated channel-state process and j_e the canonical value of the estimate. The channel estimation procedure satisfies (A3.5). The controls will be a function of the queue state and the current estimate of the channel state. When the estimate is j_e, the rate of transmission per unit power is now written as $\bar{\lambda}_i^d(j_e)$, since the basic bit interval must be based on the estimate.

Recall that j is vector-valued; it is the state of all K connecting individual channels. It is possible that some components of the channel state are known or well estimated, but other components are not estimated at all. This possibility is covered in the following framework.

A3.5. *The channel estimation is done at the start of each scheduling interval, with no filtering (i.e., previous estimates are not used to get the current estimate). Let $\Pi(j_e|j)$ denote the probability (conditioned on the systems data to the start of the current interval, and that the current true channel state is j) that the current estimate is j_e. Let $q_i(j, j_e, u_i(j_e, \bar{x}))$ denote the probability that a packet transmitted in the current interval is accepted at the receiver, conditioned on the data to the time of completion of transmission, when the channel state is j, the estimate is j_e and queue-state vector at the start of the interval is $\bar{x}$. Suppose that $q_i(j, j_e, u_i)$ is continuous in u_i. Also, $q_i(j, j_e, 0) = 0$*

The fluid limit will have the form

$$\begin{aligned} \bar{x}_i(t) &= \bar{x}_i(0) + \int_0^t b_i^e(\bar{x}(s), u(\bar{x}))ds, \\ b_i^e(\bar{x}, u(\bar{x})) &= \bar{\lambda}_i^a \bar{v}_i^a - \sum_{j,j_e} \bar{\lambda}_i^d(j_e) u_i(j_e, \bar{x}) q_i(j, j_e, u_i(j_e, \bar{x}))\Pi(j_e|j)\Pi(j). \end{aligned} \tag{3.17}$$

Define $\Pi_e(j_e) = \sum_j \Pi(j_e|j)\Pi(j)$, the "stationary probability" that the estimate is j_e. Define the conditional probability $\Pi(j|j_e) = \Pi(j_e|j)\Pi(j)/\Pi_e(j_e)$ by Bayes' rule, and define the average probability of acceptance of a packet at the receiver

$$\bar{q}_i(j_e, \alpha_i) = \sum_j q_i(j, j_e, \alpha_i)\Pi(j|j_e).$$

Then the $b_i^e(\cdot)$ in (3.17) can be written as

$$b_i^e(\bar{x}, u(\bar{x})) = \bar{\lambda}_i^a \bar{v}_i^a - \sum_{j_e} \bar{\lambda}_i^d(j_e) u_i(j_e, \bar{x}) \bar{q}_i(j_e, u_i(j_e, \bar{x}))\Pi_e(j_e), \tag{3.18}$$

which is (3.8) with the average probability used and j_e replacing j.

Theorem 3.2. *Assume* (A3.1), (A3.2), (A3.4), (A3.5), *and the model of Example* 2.1 *with continuous $u_i(j, \cdot)$ and $u_i(j, 0) = 0$. Then $\{\bar{x}^n(\cdot)\}$ is tight and the limit of any weakly convergent subsequence satisfies* (3.17) *and* (3.18).

Comments on the proof. The development is similar to that of Theorem 3.1. The expression (3.9) is replaced by

$$\frac{1}{n}\sum_{l=0}^{t/\delta_n} I_{i,l\delta_n}^{d,n} \left[I_{i,l\delta_n}^{r,n} - q_i(j, j_e, u_i(j_e, \bar{x}^n[l\delta_n])) \right] I_{\{L^n[l\delta_n]=j, L^{e,n}[l\delta_n]=j_e\}}. \tag{3.19}$$

By (A3.5), the summands are still martingale differences. Thus, using the procedure in Theorem 3.1, the analog of (3.10) is seen to be

$$\frac{1}{n}\sum_{l=0}^{t/\delta_n} \sum_{j,j_e} I_{i,l\delta_n}^{d,n} q_i(j, j_e, u_i(j_e, \bar{x}^n[l\delta_n])) I_{\{L^n[l\delta_n]=j, L^{e,n}[l\delta_n]=j_e\}}. \tag{3.20}$$

By (A3.5) and another "martingale" argument the indicator function $I_{\{L^n[l\delta_n]=j, L^{e,n}[l\delta_n]=j_e\}}$ can be (asymptotically) replaced by $I_{\{L^n[l\delta_n]=j\}}\Pi(j_e|j)$ without changing the limits. This yields

$$\frac{1}{n}\sum_{l=0}^{t/\delta_n}\sum_{j,j_e} I^{d,n}_{i,l\delta_n} q_i(j, j_e, u_i(j_e, \bar{x}^n[l\delta_n])) I_{\{L^n[l\delta_n]=j\}}\Pi(j_e|j). \tag{3.21}$$

The rest of the development is similar to that of Theorem 3.1.

3.2 Example 2.2. Known or Incompletely Known Channel State.

The development is nearly identical to that of Theorems 3.1 or 3.2. Arbitrary combinations of the two canonical forms can be treated similarly. The model in the following theorem is that of the Extension to Example 2.2, where the rate of transmission is proportional to the power and the effective rate per unit time is given by (2.4). Thus, there are two controls, which are the power and the time allocations.

Theorem 3.3. *Assume* (A3.1), (A3.2), (A3.4), (A3.5), *the model of the extension to Example* 2.2, *and* (2.2). *Let* $v_i(j_e,\cdot)$ *and* $u_i(j_e,\cdot)$ *be continuous with* $u_i(j,0) = v_i(j,0) = 0$. *Then set* $\{\bar{x}^n(\cdot)\}$ *is tight and the limit of any weakly convergent subsequence satisfies* ($b_i^e(\cdot)$ *is redefined*) *here*

$$\begin{aligned} \bar{x}_i(t) &= \bar{x}_i(0) + \int_0^t b_i^e(\bar{x}(s), v(\bar{x}(s)), u(\bar{x}(s)))ds \\ b_i^e(\bar{x}, v(\bar{x}), u(\bar{x})) & \\ &= \bar{\lambda}_i^a \bar{v}_i^a - \sum_{j,j_e} \bar{\lambda}_i^d(j_e) u_i(j_e,\bar{x}) v_i(j_e,\bar{x}) q_i(j, j_e, u_i(j_e,\bar{x}))\Pi(j_e|j)\Pi(j), \end{aligned} \tag{3.22}$$

or, equivalently,

$$b_i^e(\bar{x}, v(\bar{x}), u(\bar{x})) = \bar{\lambda}_i^a \bar{v}_i^a - \sum_{j_e} \bar{\lambda}_i^d(j_e) u_i(j_e,\bar{x}) v_i(j_e,\bar{x}), \bar{q}_i(j_e, u_i(j_e,\bar{x}))\Pi_e(j_e). \tag{3.23}$$

Comments on the proof. The approximations (3.19)–(3.21) still hold. Again, the coefficients of $I^{d,n}_{i,l\delta_n}$ in (3.19), (3.20) for $l\delta_n$ in the μth scheduling interval depend only on the queue state and channel state estimate at the beginning of that interval, and not otherwise on the index l. For fixed j, the sum of the terms $I^{d,n}_{i,l\delta_n}/n$ in that scheduling interval equals

$$\bar{\lambda}_i^d(j_e) v_i(j_e,\bar{x}) u_i(j_e,\bar{x}) \bar{\Delta}^{s,n},$$

where $\bar{x}$ is the queue state and j_e the channel state estimate at the beginning

of the interval. Thus, in the present case (3.11) is replaced by

$$\delta_n \sum_{l=0}^{t/\delta_n} \sum_{j,j_e} \bar{\lambda}_i^d(j_e) q_i(j, j_e, u_i(j_e, \bar{x}^n[l\delta_n])) u_i(j_e, \bar{x}^n[l\delta_n]) \\ \times v_i(j_e, \bar{x}^n[l\delta_n]) \Pi(j_e|j) I_{\{L^n[l\delta_n]=j\}}. \tag{3.24}$$

The rest of the development is similar to that of Theorems 3.1 and 3.2.

Comments on the generality of the approach. Theorem 3.3 includes Theorems 3.1 and 3.2. The forms (3.19)–(3.21) for the number of packets successfully transmitted is the same for all cases. If the channel state is known, then just let $j_e = j$. These expressions simply represent counting of the transmitted and accepted packets, whether centered about the acceptance probability or not. They can be used as long as the acceptance probability depends only on the channel state, its estimate, and the applied power, whatever the physical system happens to be. Thus, one needs only to evaluate the form (3.21) to get the limit process. As in the theorems, this form is evaluated for each scheduling interval separately. Since the coefficient of the $I_{i,l\delta_n}^{d,n}$ is constant over the scheduling interval, it is only the number of nonzero $I_{i,l\delta_n}^{d,n}$ in that interval that need to be computed. But this is just the rate of transmission of packets, which depends only on the choices made at the start of the interval. With this understanding, various combinations of Examples 2.1 and 2.2 are possible, For example, the subintervals in Example 2.2 can overlap each other.

4 Stability: Assumptions: Example 2.1

Owing to the fact that the queue length processes are not Markov, the classical Liapunov function methods [4, 7] cannot be used directly. The perturbed Liapunov function method of the form in [8, 10] is a powerful method for non-Markovian problems of the types that arise in this paper. The method starts with a classical Liapunov function for the fluid limit, and then adds appropriate perturbations to "average" the non-Markovian "noise."

The assumptions that will be used to define the perturbations will be stated and discussed in this section. It will be seen that the requirements are modest. We will concentrate on Example 2.1, when the channel state is known. The minor changes for the other cases will be discussed at the end of the next section. The actual stability proof is given in the next section. The service processes are determined by the system structure in Example 2.1 or 2.2.

We choose a particular set of conditions, defined by (A4.1), for the arrival processes. These are weaker than (A3.2), but are only illustrative of the possibilities. The form is chosen to cover "steady" arrival streams as well as large bursts. The form is flexible and allows numerous variations of interest, as noted in the discussion below (A4.3).

The smoothness condition on $u_i(j, \bar{x})$ in (A4.3) is a technical requirement for the general problem and can be weakened under additional (but still reasonable)

assumptions on the problem. This is seen in the remarks at the end of Section 6, where we show that the smoothness condition can be dropped for the problem of concern there.

When we say that a sum is well-defined in (A4.1) and (A4.2), we mean that the sequence $\sum_{l=k}^{M}$ converges boundedly as $M \to \infty$, uniformly in ω, with probability one, for each (k, n). More generally, we could use discounted sums of the type discussed at the end of Example 4.2. We also note that the development could be extended to allow the arrival rates, mean batch sizes, and service rates to depend on the queue state. For example, replace $\bar{\lambda}_i^a$ by $\bar{\lambda}_i^a(\bar{x}^n[\mu\Delta^{s,n}])$ during the μth scheduling interval, where $\bar{\lambda}_i^a(\cdot)$ is bounded and continuous, and change the stability condition (A5.6) accordingly.

A4.1. *The channel-state process and the data arrival process are mutually independent. For each i, k, the sum*

$$C_{i,k\delta_n}^{a,n} = \frac{1}{n} \sum_{l=k}^{\infty} E_{k\delta_n}^n \left[v_{i,l\delta_n}^{a,n} I_{i,l\delta_n}^{a,n} - n\delta_n \bar{\lambda}_i^a \bar{v}_i^a \right], \tag{4.1}$$

is well-defined, and bounded uniformly in k, ω, with probability one.

A4.2. *For each i, j, k, the sum*

$$\begin{aligned} & C_{k\delta_n}^{d,0,n}(j) = \\ & \quad -\delta_n \sum_{l=k}^{\infty} E_{k\delta_n}^n \left[I_{\{L^n[l\delta_n]=j\}} - \Pi(j) \right] = -\delta_n \sum_{l=k}^{\infty} E_{k\delta_n}^n \left[I_{\{L[n^\nu l\delta_n]=j\}} - \Pi(j) \right] \end{aligned} \tag{4.2}$$

is well-defined and, as $n \to \infty$, it goes to zero, uniformly in k, ω, with probability one.

A4.3. *$q_i(j, \cdot)$ and $u_i(j, \cdot)$ have bounded and uniformly continuous first order partial derivatives.*

Discussion of the process $C_{i,k\delta_n}^{a,n}$ for the arrivals. Let us examine $C_{i,k\delta_n}^{a,n}$ more closely to understand why our requirement on its value is reasonable. Consider queue i. The following examples are illustrative, but not exhaustive.

Example 4.1. First, suppose that the unscaled packet arrival process is compound Poisson with rate $n\bar{\lambda}_i^a$, with the batch sizes having a uniformly bounded variance and mean $\bar{v}_i^a$. Approximate to discrete time so that rate $n\bar{\lambda}_i^a$, means that the conditional probability of a single arrival on any single interval $[l\delta_n, l\delta_n + \delta_n)$ is $\delta_n n \bar{\lambda}_i^a$, with multiple arrivals not possible. Then, owing to the use of the conditional expectation, each of the summands in (4.1) is zero. Under this Poisson condition, (A3.2) also holds. Now, let the rate for the unscaled process be $\bar{\lambda}_i^a$, with batch sizes $nv_{i,l\delta_n}^{a,n}$ for a batch arriving at $l\delta_n$. Let $v_{i,l\delta_n}^{a,n}$ be bounded with mean $\bar{v}_i^a$. Then, the summands in (4.1) are still zero, but (A3.2) no longer

holds. Any limit of $\bar{x}^n(\cdot)$ would be subject to a Poisson driving process. Such jumps are one way of modeling burstiness: The input bursts can be of the order of the speed of the system. Clearly intermediate Poisson cases are possible.

Example 4.2. To exhibit another class of examples, define $\bar{\Delta}_i^a = 1/\bar{\lambda}_i^a$. Since $\bar{\lambda}_i^a \bar{v}_i^a$ is merely a *centering* constant for the *entire* sequence, the actual mean values or rates can vary with time (say, being periodic, etc.). Since the arrival rate is $O(n)$, scale the interarrival times by defining a quantity $\Delta_{i,l}^{a,n}$ such that the interarrival times are $\Delta_{i,l}^{a,n}/n$. Fix k and let $\mu_{i,1}^{k,n}\delta_n$ and $\mu_{i,2}^{k,n}\delta_n$ denote the times of the first two arrivals to queue i at or after time $k\delta_n$. Consider the part of $C_{i,k\delta_n}^{a,n}$ given by

$$E_{k\delta_n}^n \sum_{l=\mu_{i,1}^{,n}+1}^{\mu_{i,2}^{k,n}} \left[v_{i,l\delta_n}^{a,n} I_{i,l\delta_n}^{a,n} - n\delta_n \bar{\lambda}_i^a \bar{v}_i^a \right].$$

This equals

$$E_{k\delta_n}^n \left[v_{i,\mu_{i,2}^{k,n}\delta_n}^{a,n} - (\mu_{i,2}^{k,n} - \mu_{i,1}^{k,n}) n\delta_n \bar{\lambda}_i^a \bar{v}_i^a \right]. \tag{4.3}$$

Next, for the moment, suppose that the interarrival times are mutually independent and identically distributed, with finite second moments, and mean $\bar{\Delta}_i^a/n$, and that the batch sizes are mutually independent and independent of the set of arrival times, and also have bounded second moments. Then the conditional expectation (4.3) equals zero w.p.1, since $E_{k\delta_n}^n(\mu_{i,2}^{k,n} - \mu_{i,1}^{k,n})\delta_n = \bar{\Delta}_i^a/n$ Obviously $\mu_{i,1}^{k,n}\delta_n$ and $\mu_{i,2}^{k,n}\delta_n$ can be any two successive arrival times with the same result. Thus, under the independence assumption, the sum in $C_{i,k\delta_n}^{a,n}$ is just composed of the terms up to the time of the first arrival at or after time $k\delta_n$, namely,

$$\bar{v}_i^a E_{k\delta_n}^n \left[1 - (\mu_{i,1}^{k,n} - k) \bar{\lambda}_i^a n\delta_n \right] = \bar{v}_i^a E_{k\delta_n}^n \left[1 - \frac{(\mu_{i,1}^{k,n} - k) n\delta_n}{\bar{\Delta}_i^a} \right], \tag{4.4}$$

where $E_{k\delta_n}^n(\mu_{i,1}^{k,n} - k)\delta_n$ is just the conditional expectation of the mean time to the next arrival to queue i at or after time $k\delta_n$, given the data to time $k\delta_n$. The quantity (4.4) is bounded uniformly in k, n, under the assumptions given above on the independence and the moments.

Now, suppose that the interarrival times are correlated, still with mean $\bar{\Delta}_i^a/n$, but retain the independence assumption on the set of batch sizes. Let $\mu_{i,l}^{k,n}\delta_n, l = 1, \ldots,$ denote the sequence of arrival times to queue i at or after time $k\delta_n$. Then

$$E_{k\delta_n}^n \left[1 - (\mu_{i,l+1}^{k,n} - \mu_{i,l}^{k,n}) n\delta_n \bar{\lambda}_i^a \right] = E_{k\delta_n}^n \left[1 - \frac{\Delta_{i,l}^a}{\bar{\Delta}_i^a} \right].$$

Then, grouping terms, we see that the *sum* in $C^{a,n}_{i,k\delta_n}$ is just (4.4) plus the series

$$\sum_{l:\mu^{k,n}_{i,l}\geq\mu^{k,n}_{i,1}}^{\infty} E^n_{k\delta_n}\left[1-\frac{\Delta^a_{i,l+1}}{\bar{\Delta}^a_i}\right]v^{a,n}_{i,l+1}.$$

Owing to the conditional expectation, this sum is well-defined and bounded uniformly in n, ω, under broad mixing conditions. So we see that (A4.1) is not restrictive.

Discounted forms. Suppose that the batches in Example 4.2 cycle, alternating between 1 and 2. Then the sum $\sum_{l=k}^{M}$ of the summands in (4.1) will not converge, but oscillate. This can be dealt with by using a discounted form of $C^{a,n}_{i,k\delta_n}$ such as

$$\begin{aligned}E^n_{k\delta_n}\sum_{l=k}^{\mu^{k,n}_{i,1}} e^{-\delta_n(l-k)}&\left[v^{a,n}_{i,l\delta_n}I^{a,n}_{i,l\delta_n}-n\delta_n\bar{\lambda}^a_i\bar{v}^a_i\right]\\ &+\sum_{l:\mu^{k,n}_{i,l}\geq\mu^{k,n}_{i,1}}^{\infty} E^n_{k\delta_n}e^{-\delta_n(\mu^{k,n}_{i,l}-k)}\left[1-\frac{\Delta^a_{i,l+1}}{\bar{\Delta}^a_i}\right]v^{a,n}_{i,l+1}.\end{aligned}$$

This last expression simply discounts the summands up to the first arrival at or after $k\delta_n$, and then discounts the rest according to the beginning of the associated interarrival interval. With a little extra algebra, such forms can be used to extend (A4.1) and (A4.2), but we prefer to avoid the extra detail here. See [10, 13] for other uses of such discounting in convergence theorems.

Example 4.3. Consider a situation where "arrival events" to queue i occur at integral multiples of $c_i > 0$, and an "arrival event" at time $\mu c_i, \mu = 1, 2, \ldots,$ has value $n\nu^{a,n}_{i,\mu c_i}$, where the $\nu^{a,n}_{i,\mu c_i}$ have mean $\bar{v}^a_i$. Suppose that the $\nu^{a,n}_{i,\mu c_i}$ are m-dependent. By "arrival event" at μc_i of value $n\nu^{a,n}_{i,\mu c_i}$, we mean that this amount of data is ready to be transferred to the queue at that time, but due to bandwidth limitations it arrives at a rate nd_i over an interval of time $\nu^{a,n}_{i,\mu c_i}/d_i$. Let $\nu^{a,n}_{i,\mu c_i} \leq d_i c_i$. Define $\bar{\lambda}^a_i = 1/c_i$ and let $v^{a,n}_{i,k\delta_n}$ denote the amount that actually arrives at time $k\delta_n$. Then the "tail" of (4.1) is zero and (4.1) is well-defined and bounded. This example can readily be generalized, the main point being that bandwidth limitations and input control mechanisms will usually assure a maximum (true, not average) arrival rate of $O(n)$.

Discussion of the process (4.2) for the departures. Suppose that the process $L(\cdot)$ is a finite-state ergodic Markov chain in continuous time with stationary probabilities $\Pi(j)$. Then there are positive constants C, λ such that for $l \geq k$

$$\left|E^n_{k\delta_n}I_{\{L(n^\nu l\delta_n)=j\}}-\Pi(j)\right|\leq Ce^{-\lambda n^\nu(l-k)\delta_n}.$$

Then (4.2) is bounded by

$$C\frac{\delta_n}{1-e^{\lambda n^\nu \delta_n}} \approx C\frac{1}{\lambda n^\nu}.$$

This bound also holds for a large set of semi-Markov processes. More generally, $L(\cdot)$ could be a mixing process for which $C^{d,0,n}_{k\delta_n}(j)$ goes to zero uniformly in (k,ω) as $n\to\infty$.

Auxiliary remarks concerning (A4.2). In the next section, (A4.2) will be used in connection with the process

$$C^{d,1,n}_{i,k\delta_n}(j) = -\frac{1}{n}\sum_{l=k}^{\infty} E^n_{k\delta_n}\left[I^{d,n}_{i,l\delta_n} I^{r,n}_{i,l\delta_n} - n\delta_n r_i(j,\bar{x}^n[l\delta_n])\right] I_{\{L^n[l\delta_n]=j\}}, \tag{4.5}$$

where we define

$$r_i(j,\bar{x}) = \bar{\lambda}^d_i(j) q_i(j,u_i(j,\bar{x}))u_i(j,\bar{x}). \tag{4.6}$$

Fix k and redefine $\mu^{k,n}_{i,1}\delta_n$ and $\mu^{k,n}_{i,2}\delta_n$ be the times of the first two completions of packet *transmissions* from queue i at or after time $k\delta_n$ that were accepted at the receiver. Consider the expression

$$E^n_{[k\delta_n]} \sum_{l=\mu^{k,n}_{i,1}+1}^{\mu^{k,n}_{i,2}} \left[I^{d,n}_{i,l\delta_n} I^{r,n}_{i,l\delta_n} - n\delta_n r_i(j,\bar{x}^n[l\delta_n])\right] I_{\{L^n[l\delta_n]=j\}}. \tag{4.7}$$

For the moment, suppose that $q_i(j,u_i(j,\bar{x}))=1$ for all $i,j,\bar{x}$. Then the expected time interval between completed packet transmissions in the current scheduling interval (conditioned on the data to $[k\delta_n]$) is just the inverse of the rate and is

$$\frac{1}{n\bar{\lambda}^d_i(j)u^n_i(j,\bar{x}^n[l\delta_n])}. \tag{4.8}$$

This is

$$Q^n_{i,l\delta_n}(j) = \frac{1}{n\delta_n\bar{\lambda}^d_i(j)u^n_i(j,\bar{x}^n[l\delta_n])} \tag{4.9}$$

multiples of the basic measurement unit δ_n. Hence

$$E^n_{[k\delta_n]}\left(\mu^{k,n}_{i,2}-\mu^{k,n}_{i,1}\right) n\delta_n\bar{\lambda}^d_i(j)u_i(j,\bar{x}^n[l\delta_n]) = \frac{E^n_{[k\delta_n]}\left(\mu^{k,n}_{i,2}-\mu^{k,n}_{i,1}\right)}{Q^n_{i,l\delta_n}(j)} = 1. \tag{4.10}$$

The sum over the indicator functions $I^{d,n}_{i,l\delta_n} I^{r,n}_{i,l\delta_n}$ in (4.7) is also unity. Hence (4.7) equals zero. The result is similar if $\mu^{k,n}_{i,1}\delta_n$ and $\mu^{k,n}_{i,2}\delta_n$ are the times of any successive completions of packet transmissions in the scheduling interval. We can conclude that for all k,ω, the sum $\sum_{l=\mu^{k,n}_{i,1}+1}^{\infty}$ part of (4.5) is zero. Hence,

$$\left|C^{d,1,n}_{i,k\delta_n}(j)\right| \le C_1/n \tag{4.11}$$

for some constant C_1.

Now, admit the possibility of rejections, but with $q_i(j, u_i(j, \bar{x}))$ being either greater than some positive number or else zero, corresponding to no power. Redefine $\mu_{i,1}^{k,n}\delta_n$ and $\mu_{i,2}^{k,n}\delta_n$ to be the times of the first two completions of packet transmissions from queue i at or after time $k\delta_n$ *that were accepted* at the receiver. The time between completed transmissions in the scheduling interval is still (4.8). Thus the left side of (4.10) is just the conditional mean number of packets transmitted per acceptance at the receiver. Since this equals $1/q_i(j, u_i(j, \bar{x}^n[l\delta_n]))$, (4.7) still equals zero.

We will also have use for the process defined by (see (5.4))

$$\delta_n r_i(j, \bar{x}^n[k\delta_n]) \sum_{l=k}^{\infty} E_{k\delta_n}^n \left[\Pi(j) - I_{\{L^n(l\delta_n)=j\}} \right], \tag{4.12}$$

where $r_i(j, \cdot)$ was defined in (4.6). For future use, define the replacement for $r_i(j, \cdot)$ when the channel is only partially known:

$$r_i^e(j, j_e, \bar{x}) = \bar{\lambda}_i^d(j_e) q_i(j, j_e, u_i(j_e, \bar{x})) u_i(j_e, \bar{x}). \tag{4.13}$$

Time required for channel estimation. Suppose that part of the scheduling interval is taken by estimation of the channel state, and that no actual packet transmissions can take place then. The rates of concern are over the entire scheduling interval. Thus, without changing the end result, we can account for the time required for estimation by simply adjusting the rates appropriately. [I.e., effectively spreading the estimation time over the scheduling interval.]

5 Stability Proofs

5.1 Example 2.1 With Known Channel State

The form of the perturbed Liapunov function. Let $V(\cdot)$ be a Liapunov function for the fluid model (3.8) under a chosen control $u(j, \bar{x})$ in that (A5.1) and (A5.6) hold. The actual Liapunov function which is used for the physical problem will differ by a small perturbation. The stability analysis will use (A4.1)–(A4.3) in addition to the following assumptions. If $V(\bar{x})$ is a polynomial, which is the form used in the next section, then the cases of Examples 4.1–4.3 all hold. The case where the actual number arriving in any time interval is bounded by n times the interval, modulo a small error, is also covered (assuming that (A4.1) holds). This latter form would hold if the connection to the queue of the arrival process, however bursty, was subject to a either a token bank type controller, or to bandwidth limitations of $O(n)$. (A5.4)–(A5.6) are to hold for the chosen controls. Condition (A5.8) and the last part of (A5.2) are needed since we ultimately work with the scheduling intervals.

The conditions might seem formidable at first sight. But they are reasonable, as seen from Examples 4.1–4.3 and the results in the next section. Recall the definition of stability, uniformly for large n, given in the Introduction.

A5.1. *$V(\cdot)$ is a continuous nonnegative real-valued function of $\bar{x}$ which goes to infinity as $\bar{x} \to \infty$. Its partial derivatives up to second order are continuous. The gradient is denoted by $V_{\bar{x}}(\cdot)$ and the Hessian by $V_{\bar{x}\bar{x}}(\cdot)$.*

A5.2. *The rate of arrivals is $O(n)$ in the following sense. There are constants C_i such that (all with probability one for each k, n, μ),*

$$\begin{aligned} \left[\bar{A}_i^n(k\delta_n + \delta_n) - \bar{A}_i^n(k\delta_n)\right] &\leq C_1, \\ nE_{k\delta_n}^n \left[\bar{A}_i^n(k\delta_n + \delta_n) - \bar{A}_i^n(k\delta_n)\right] &\leq C_2, \\ nE_{k\delta_n}^n \left[\bar{A}_i^n(k\delta_n + \delta_n) - \bar{A}_i^n(k\delta_n)\right]^2 &\leq C_3, \\ E_{\mu\bar{\Delta}^{s,n}}^n \left[\bar{A}_i^n(\mu\bar{\Delta}^{s,n} + \bar{\Delta}^{s,n}) - \bar{A}_i^n(\mu\bar{\Delta}^{s,n})\right]^2 &\leq C_4\bar{\Delta}^{s,n}. \end{aligned}$$

A5.3. *For each i and positive $\bar{\rho}$ and $|\rho| \leq \bar{\rho} < \infty$,*

$$\limsup_{|\bar{x}|\to\infty} \frac{|V_{\bar{x}_i}(\bar{x}+\rho)|}{|V_{\bar{x}_i}(\bar{x})|} < \infty,$$

$$\limsup_{|\bar{x}|\to\infty} \frac{|V_{\bar{x}\bar{x}}(\bar{x}+\rho)|}{|V_{\bar{x}\bar{x}}(\bar{x})|} < \infty.$$

A5.4.
$$\lim_{|\bar{x}|\to\infty} \frac{|V_{\bar{x}\bar{x}}(\bar{x})|}{|V'_{\bar{x}}(\bar{x})b(\bar{x},u(\bar{x}))|} = 0.$$

A5.5.
$$\limsup_{|\bar{x}|\to\infty} \frac{|V_{\bar{x}}(\bar{x})|}{|V'_{\bar{x}}(\bar{x})b(\bar{x},u(\bar{x}))|} < \infty.$$

A5.6.
$$\limsup_{|\bar{x}|\to\infty} V'_{\bar{x}}(\bar{x})b(\bar{x},u(\bar{x})) < 0.$$

A5.7.
$$\lim_{|\bar{x}|\to\infty} \frac{|V_{\bar{x}}(\bar{x})|}{V(\bar{x})} = 0.$$

A5.8. *Define $\Delta\bar{A}^n(\mu\bar{\Delta}^{s,n}) = \bar{A}^n(\mu\bar{\Delta}^{s,n}+\bar{\Delta}^{s,n}) - \bar{A}^n(\mu\bar{\Delta}^{s,n})$. For some $C_5 < \infty$,*

$$\limsup_{|\bar{x}|\to\infty} \sup_{\mu} \frac{E_{\mu\Delta^{s,n}}^n \left|V_{\bar{x}_i}\left(\bar{x} + \Delta\bar{A}^n(\mu\bar{\Delta}^{s,n})\right)\right| \left|\Delta\bar{A}^n(\mu\bar{\Delta}^{s,n})\right|}{|V_{\bar{x}_i}(\bar{x})|} \leq C_5\bar{\Delta}^{s,n},$$

Define $\delta\bar{A}^n_{k\delta_n} = \bar{A}^n(k\delta_n) - \bar{A}^n[k\delta_n]$. *Then for each* i

$$\limsup_{|\bar{x}|\to\infty} \sup_k \frac{E^n_{[k\delta_n]}\left|V_{\bar{x}\bar{x}}(\bar{x}+\delta\bar{A}^n_{k\delta_n})\right|}{|V_{\bar{x}\bar{x}}(\bar{x})|} < \infty.$$

There is $c_0(\bar{\Delta}^{s,n})$ which goes to zero as $\bar{\Delta}^{s,n} \to 0$ such that

$$\limsup_{|\bar{x}|\to\infty} \sup_k \frac{E^n_{[k\delta_n]}\left|V_{\bar{x}_i}(\bar{x}+\delta\bar{A}^n_{k\delta_n}) - V_{\bar{x}_i}(\bar{x})\right|}{|V_{\bar{x}_i}(\bar{x})|} \le c_0(\bar{\Delta}^{s,n}).$$

See also the remarks at the end of Section 6, concerning discontinuous controls.

Theorem 5.1. *Assume the model of Example* 2.1, (2.1), (A3.1), (A3.3), (A4.1)–(A4.3) *and* (A5.1)–(A5.8) . *Let the channel state be known. Then* $\bar{x}^n(\cdot)$ *is stable, uniformly for large* n.

Proof. The proof proceeds as usual in stability studies. One defines a Liapunov function and then shows that it has the required supermartingale property for large $\bar{x}$. Since the queue-state process is not Markovian, the perturbed Liapunov function method as described in [8] will be used. We start by using the Liapunov function $V(\cdot)$ for the fluid approximation and then "correct" it to eliminate undesirable terms in its expansion. Since arrivals and departures from the queues take place only at integral multiples of δ_n, we start by looking at the queues at those times. Ultimately, we will need to look at the queues at the starting times of the scheduling intervals, since those are the only times at which the controls can change.

Define

$$\Theta^n_{i,k} = v^{a,n}_{i,k\delta_n} I^{a,n}_{i,k\delta_n} + I^{d,n}_{i,k\delta_n}, \qquad \Theta^n_k = \sum_i \Theta^n_{i,k}.$$

A truncated Taylor series expansion yields

$$\begin{aligned} &E^n_{k\delta_n} V(\bar{x}^n(k\delta_n+\delta_n)) - V(\bar{x}^n(k\delta_n)) = \\ &\quad \frac{1}{n} E^n_{k\delta_n} \sum_i V_{\bar{x}_i}(\bar{x}^n(k\delta_n)) \left[v^{a,n}_{i,k\delta_n} I^{a,n}_{i,k\delta_n} - I^{d,n}_{i,k\delta_n} I^{r,n}_{i,k\delta_n}\right] \\ &\quad + \frac{1}{n^2} O(1) E^n_{k\delta_n} \left|V_{\bar{x}\bar{x}}(\bar{x}^n(k\delta_n) + \theta^n_{k\delta_n}/n)\right| \sum_i \left[v^{a,n}_{i,k\delta_n} I^{a,n}_{i,k\delta_n} - I^{d,n}_{i,k\delta_n} I^{r,n}_{i,k\delta_n}\right]^2, \end{aligned} \tag{5.1}$$

where $|\theta^n_{k\delta_n}| \le \Theta^n_k$. The first order terms in (5.1) depend on the random indicator functions of the arrival, service and acceptance events, as well as on the random batch sizes. We will need to replace these random variables by their "averages," and this is the role of the perturbations.

For all i, j, define the Liapunov function perturbations

$$\delta V^{a,n}_i(k\delta_n) = V_{\bar{x}_i}(\bar{x}^n(k\delta_n)) C^{a,n}_{i,k\delta_n}, \tag{5.2}$$

$$\delta V_i^{d,1,n}(j,k\delta_n) = V_{\bar{x}_i}(\bar{x}^n(k\delta_n))C_{i,k\delta_n}^{d,1,n}(j), \tag{5.3}$$

$$\delta V_i^{d,0,n}(j,k\delta_n) = V_{\bar{x}_i}(\bar{x}^n(k\delta_n))r_i(j,\bar{x}^n[k\delta_n])C_{k\delta_n}^{d,0,n}(j), \tag{5.4}$$

where $r_i(\cdot)$ is defined in (4.6). Define the "perturbed" Liapunov function

$$\begin{aligned} V^n(k\delta_n) = V(\bar{x}^n(k\delta_n)) + \sum_i \delta V_i^{a,n}(k\delta_n) + \sum_{i,j} \delta V_i^{d,1,n}(j,k\delta_n) \\ + \sum_{i,j} \delta V_i^{d,0,n}(j,k\delta_n). \end{aligned} \tag{5.5}$$

By the definition (5.2) and that of $C_{i,k\delta_n}^{a,n}$, we can write

$$\begin{aligned} &E_{k\delta_n}^n \delta V_i^{a,n}(k\delta_n+\delta_n) - \delta V_i^{a,n}(k\delta_n) = \\ &\quad -\frac{1}{n}V_{\bar{x}_i}(\bar{x}^n(k\delta_n))E_{k\delta_n}^n \left[v_{i,k\delta_n}^{a,n} I_{i,k\delta_n}^{a,n} - n\delta_n \bar{v}_i^a \bar{\lambda}_i^a\right] \\ &\quad + O(1)E_{k\delta_n}^n \left|V_{\bar{x}\bar{x}}(\bar{x}^n(k\delta_n) + \theta_{k\delta_n}^{a,n}/n)\right| \left|C_{i,k\delta_n+\delta_n}^{a,n}\right| \frac{|\Theta_k^n|}{n}, \end{aligned} \tag{5.6}$$

where $|\theta_{k\delta_n}^{a,n}| \le \Theta_k^n$. Note that when (5.6) is added to (5.1) the part $v_{i,k\delta_n}^{a,n} I_{i,k\delta_n}^{a,n}$ of the ith component of the first order term in (5.1) is effectively replaced by the "mean value" $\delta_n \bar{v}_i^a \bar{\lambda}_i^a$. The desire for such a replacement of the random terms by the averages motivated the forms selected for $C_{i,k\delta_n}^{a,n}$ and the perturbation (5.2). Analogous considerations hold for the other perturbations.

Conditions (A4.1), the first two parts of (A5.2), and the second part of (A5.3), imply that for large $\bar{x}^n(k\delta_n)$ the last term in (5.6) is

$$O(\delta_n)\left|V_{\bar{x}\bar{x}}(\bar{x}^n(k\delta_n))\right| \tag{5.7}$$

By the second parts of (A5.3) and (A5.8), for large $\bar{x}^n[k\delta_n]$ we have (note that the conditioning in the expectation in (5.8) is on the data to time $[k\delta_n]$ and not to time $k\delta_n$)

$$O(\delta_n)E_{[k\delta_n]}^n \left|V_{\bar{x}\bar{x}}(\bar{x}^n(k\delta_n))\right| = O(\delta_n)\left|V_{\bar{x}\bar{x}}(\bar{x}^n[k\delta_n])\right|. \tag{5.8}$$

By (A5.4), this last term can be bounded by

$$\delta_n \rho(\bar{x}^n[k\delta_n])\left|V_{\bar{x}}'(\bar{x}^n[k\delta_n])b(\bar{x}^n[k\delta_n], u(j,\bar{x}^n[k\delta_n])\right|, \tag{5.9}$$

where $\lim_{|\bar{x}|\to\infty} \rho^n(\bar{x}) = 0$. Using, in addition, the third line of (A5.2), the last term of (5.1) can also be bounded by the form (5.7).

Expanding (5.3) and using the definition (4.5) of $C_{i,k\delta_n}^{d,1,n}(j)$ and that of $r_i(j,\bar{x})$ in (4.6) yields

$$\begin{aligned} &E_{k\delta_n}^n \delta V_i^{d,1,n}(j,k\delta_n+\delta_n) - \delta V_i^{d,1,n}(j,k\delta_n) = \\ &\quad \frac{1}{n}V_{\bar{x}_i}(\bar{x}^n(k\delta_n))E_{k\delta_n}^n \left[I_{i,k\delta_n}^{d,n} I_{i,k\delta_n}^{r,n} - n\delta_n r_i(j,\bar{x}^n[k\delta_n])\right] I_{\{L^n[k\delta_n]=j\}} \\ &\quad + O(1)E_{k\delta_n}^n \left|V_{\bar{x}\bar{x}}(\bar{x}^n(k\delta_n) + \theta_{k\delta_n}^{d,1,n}/n)\right| \left|C_{i,k\delta_n+\delta_n}^{d,1,n}(j)\right| \frac{|\Theta_k^n|}{n}, \end{aligned} \tag{5.10}$$

where $|\theta^{d,1,n}_{k\delta_n}| \leq \Theta^n_k$. By (4.11), the first part of (A5.2), and the second part of (A5.3), for large $\bar{x}^n(k\delta_n)$ the last term of (5.10) can be represented by the form (5.7). When (5.10) is added to (5.1) we see that the we see that the terms $I^{d,n}_{i,k\delta_n} I^{r,n}_{i,k\delta_n}$ are replaced by the "averaged" value $\delta_n r_i(j, \bar{x}^n[k\delta_n])$, when the channel state is given.

We need only "average out" the $I_{\{L^n[k\delta_n]=j\}}$ in (5.10), the terms that are due to the random channel variations. This will be facilitated by use of the perturbation (5.4). Expanding (5.4) and using the definition (4.2) yields

$$\begin{aligned} &E^n_{k\delta_n} \delta V^{d,0,n}_i(j, k\delta_n + \delta_n) - \delta V^{d,0,n}_i(j, k\delta_n) \\ &\quad = \delta_n V_{\bar{x}_i}(\bar{x}^n(k\delta_n)) r_i(j, \bar{x}^n[k\delta_n]) \left[E^n_{k\delta_n} I_{\{L^n[k\delta_n]=j\}} - \Pi(j)\right] + \epsilon^n_{i,k\delta_n}(j), \end{aligned} \tag{5.11}$$

where the "error term" $\epsilon^n_{i,k\delta_n}(j)$ is

$$\begin{aligned} &E^n_{k\delta_n} \left[V_{\bar{x}_i}(\bar{x}^n(k\delta_n + \delta_n)) r_i(j, \bar{x}^n[k\delta_n + \delta_n]) - V_{\bar{x}_i}(\bar{x}^n(k\delta_n)) r_i(j, \bar{x}^n[k\delta_n])\right] \\ &\quad \times C^{d,0,n}_{k\delta_n+\delta_n}(j). \end{aligned} \tag{5.12}$$

Rewrite (5.12) as

$$\begin{aligned} &E^n_{k\delta_n} \left[V_{\bar{x}_i}(\bar{x}^n(k\delta_n + \delta_n)) - V_{\bar{x}_i}(\bar{x}^n(k\delta_n))\right] r_i(j, \bar{x}^n[k\delta_n]) C^{d,0,n}_{k\delta_n+\delta_n}(j) \\ &\quad + E^n_{k\delta_n} V_{\bar{x}_i}(\bar{x}^n(k\delta_n + \delta_n)) \left[r_i(j, \bar{x}^n[k\delta_n + \delta_n]) - r_i(j, \bar{x}^n[k\delta_n])\right] C^{d,0,n}_{k\delta_n+\delta_n}(j). \end{aligned} \tag{5.13}$$

Using (A5.1), the first two parts of (A5.2), the second part of (A5.3), and the fact that $C^{d,0,n}_{k\delta_n+\delta_n}(j)$ goes to zero uniformly in k, ω, as $n \to \infty$, yields that for large $\bar{x}^n(k\delta_n)$ the first term of (5.13) is bounded by (5.7).

Summing (5.1), (5.6), (5.10), (5.11), and summing over the canonical channel state j and queue index i, yields

$$\begin{aligned} &E^n_{k\delta_n} V^n(k\delta_n + \delta_n) - V^n(k\delta_n) \\ &\quad \leq \delta_n \sum_i V_{\bar{x}_i}(\bar{x}^n(k\delta_n)) b_i(\bar{x}^n[k\delta_n], u(\bar{x}^n[k\delta_n]) + \epsilon^n_{k\delta_n}, \end{aligned} \tag{5.14}$$

where $\epsilon^n_{k\delta_n}$ is the sum of (5.7) and the second term of (5.13). In (5.14), $V_{\bar{x}_i}(\cdot)$ and its derivatives are evaluated at $\bar{x}^n(k\delta_n)$, but the control terms are evaluated at $\bar{x}^n[k\delta_n]$.

The second term of (5.13) is zero unless a new scheduling interval begins at time $k\delta_n + \delta_n$. The smoothness condition (A4.3) together with the first parts of all of (A5.2), (A5.3), and (A5.8), and the fact that $C^{d,0,n}_{k\delta_n+\delta_n}(j) \to 0$ uniformly in k, ω, implies that for large $\bar{x}^n[k\delta_n]$ the conditional expectation $E^n_{[k\delta_n]}$ acting on the second term of (5.13) is bounded by

$$\epsilon'_n I^n_{k\delta_n} \left|V_{\bar{x}_i}(\bar{x}^n[k\delta_n])\right| \bar{\Delta}^{s,n}, \tag{5.15}$$

where ϵ'_n is a nonrandom sequence that goes to zero as $n \to \infty$ and $I^n_{k\delta_n}$ is the indicator function of the event that a new scheduling interval starts at $k\delta_n + \delta_n$.

Sum (5.15) over $k\delta_n$ in the μth scheduling interval: i.e over k such that $[k\delta_n] = \mu\bar{\Delta}^{s,n}$. This yields

$$
\begin{aligned}
&E^n_{\mu\bar{\Delta}^{s,n}} V^n(\mu\bar{\Delta}^{s,n} + \bar{\Delta}^{s,n}) - V^n(\mu\bar{\Delta}^{s,n}) \\
&\quad \le \delta_n E^n_{\mu\bar{\Delta}^{s,n}} \delta_n \sum_{k:[k\delta_n]=\mu\bar{\Delta}^{s,n}} V'_{\bar{x}}(\bar{x}^n(k\delta_n)) b(\bar{x}^n(\mu\bar{\Delta}^{s,n}), u(\bar{x}^n(\mu\bar{\Delta}^{s,n}))) \\
&\qquad + E^n_{\mu\bar{\Delta}^{s,n}} \sum_{k:[k\delta_n]=\mu\bar{\Delta}^{s,n}} \epsilon^n_{k\delta_n}.
\end{aligned} \tag{5.16}
$$

Then use the third part of (A5.8) and the first parts of (A5.2) and (A5.3) to replace the $V_{\bar{x}_i}(\bar{x}^n(k\delta_n))$ in (5.16) by $V_{\bar{x}_i}(\bar{x}^n(\mu\bar{\Delta}^{s,n}))$, modulo an error which is bounded by $c_0(\bar{\Delta}^{s,n})|V_{\bar{x}_i}(\bar{x}^n(\mu\bar{\Delta}^{s,n}))|$ for large $\bar{x}^n(\mu\bar{\Delta}^{s,n})$. Finally, use the bounds (5.9), (5.15), and the bound (A5.5), to dominate the error terms and write, for large $\bar{x}^n(\mu\bar{\Delta}^{s,n})$ and large n,

$$
\begin{aligned}
&E^n_{\mu\bar{\Delta}^{s,n}} V^n(\bar{x}^n(\mu\bar{\Delta}^{s,n} + \bar{\Delta}^{s,n})) - V^n(\bar{x}^n(\mu\bar{\Delta}^{s,n})) \\
&\quad \le \bar{\Delta}^{s,n} V'_{\bar{x}}(\bar{x}^n(\mu\bar{\Delta}^{s,n})) b(\bar{x}^n(\mu\bar{\Delta}^{s,n}), u(\bar{x}^n(\mu\bar{\Delta}^{s,n})))/2,
\end{aligned} \tag{5.17}
$$

If the $V^n(\cdot)$ on the left hand side of (5.18) were $V(\cdot)$, then (using the fact that $V(\bar{x}) \to \infty$ as $|\bar{x}| \to \infty$) the proof would be completed, since then by (A5.6) $V(\bar{x}^n(\mu\bar{\Delta}^{s,n}))$ would have the supermartingale property for large enough n and queue-state values. In any case, the process $V^n(\mu\bar{\Delta}^{s,n})$ has the supermartingale property for large n and queue-states $\bar{x}^n(\mu\bar{\Delta}^{s,n})$. Furthermore, for some $C < \infty$,

$$
\begin{aligned}
&\left|V^n(\mu\bar{\Delta}^{s,n}) - V(\bar{x}^n(\mu\bar{\Delta}^{s,n}))\right| \\
&\quad \le C \left|V_{\bar{x}}(\bar{x}^n(\mu\bar{\Delta}^{s,n}))\right| \sum_i \left[\left|C^{a,n}_{i,k\delta_n}\right| + \sum_j \left|C^{d,0,n}_{k\delta_n}(j)\right| + \sum_j \left|C^{d,1,n}_{i,k\delta_n}(j)\right| \right].
\end{aligned}
$$

The bracketed term on the right is bounded. This and (A5.7) imply that

$$
V^n(\mu\bar{\Delta}^{s,n}) = V(\bar{x}^n(\mu\bar{\Delta}^{s,n}))(1 + c^n_1)
$$

where c^n_1 is arbitrarily small for large $\bar{x}^n(\mu\bar{\Delta}^{s,n})$ and large n. By [8, Chapter 6, Theorem 2], the stability, uniformly for large n and queue-state values follows from this, the fact that $\lim_{|\bar{x}|\to\infty} V(\bar{x}) = \infty$, and the fact that by (A5.6) there is $\epsilon > 0$ such that right hand side of (5.18) is less than $-\epsilon\bar{\Delta}^{s,n}$ for large queue-state values. ■

Example 2.1: Partially unknown channel. The development is nearly the same. In (A4.3), use $q_i(j, j_e, \cdot)$ and $u_i(j_e, \cdot)$ in lieu of $q_i(j, \cdot)$ and $u_i(j, \cdot)$, resp. The function $C^{d,1,n}_{i,k\delta_n}(j)$, defined in in (4.5), is replaced by

$$
\begin{aligned}
&C^{d,1,n}_{i,k\delta_n}(j, j_e) = \\
&-\frac{1}{n} \sum_{l=k}^{\infty} E^n_{k\delta_n} \left[I^{d,n}_{i,l\delta_n} I^{r,n}_{i,l\delta_n} - n\delta_n r^e_i(j, j_e, \bar{x}^n[l\delta_n]) \right] I_{\{L^n[l\delta_n]=j, L^{e,n}[l\delta_n]=j_e\}},
\end{aligned}
$$

where $r^e(j, j_e, \cdot)$ is defined in (4.13). In (A5.6), replace $b(\cdot)$ by the $b^e(\cdot)$ of (3.17) or (3.18). Then, under the assumptions of Theorem 5.1, but with (A3.5) replacing (A3.3), the conclusion of the theorem holds. The proof is nearly identical to that of Theorem 5.1.

5.2 Example 2.2 With Partially Known Channel State

Consider the case of Theorem 3.3, where control over the bit length was also allowed. Modify the remarks in the last paragraph of the previous subsection by defining $b^e(\cdot)$ by (3.22) or (3.23) and redefining $r_i^e(\cdot)$ to be

$$r_i^e(j, j_e, \bar{x}) = \bar{\lambda}_i^d(j_e) q_i(j, j_e, u_i(j_e, \bar{x})) v_i(j_e, \bar{x}) u_i(j_e, \bar{x}).$$

Then the conclusions of the theorem hold. Again, the proof is nearly identical to that of Theorem 5.1.

6 Examples and Sufficient Conditions

Admissible controls. The power and time allocations might be subject to constraints. For example, a minimum amount of power might be required no matter what the channel state, if the queue is not too small. There might be an upper limit on the power. The power might be constrained to a set of discrete multiples of some basic unit, as when the bit interval is controlled to be such a discrete multiple of a basic unit, and there is a lower bound on the signal to noise ratio at the receiver, if the allocated power is not zero. An *admissible power allocation* satisfies whatever constraints there are and (2.1) (and (2.2) as well where appropriate). Depending on the case, one of the following three conditions will be used.

A6.1. Example 2.1, known channel state. *There is an admissible non-state-dependent power allocation* $\{\bar{u}_i(j), i \leq K\}$, *all* j, *such that* $\sum_i \bar{u}_i(j) = \bar{u}$ *and for all* i

$$\bar{\lambda}_i^a < \sum_j \bar{\lambda}_i^d(j) \bar{u}_i(j) q_i(j, \bar{u}_i(j)) \Pi(j). \tag{6.1}$$

A6.2. Example 2.1, partially known channel state. *There is an admissible non-state-dependent power allocation* $\{\bar{u}_i(j_e), i \leq K\}$, *all* j_e, *such that* $\sum_i \bar{u}_i(j_e) = \bar{u}$ *and for all* i

$$\begin{aligned} \bar{\lambda}_i^a &< \sum_{j, j_e} \bar{\lambda}_i^d(j_e) \bar{u}_i(j_e) q_i(j, j_e, \bar{u}_i(j_e)) \Pi(j_e|j) \Pi(j) \\ &= \sum_{j_e} \bar{\lambda}_i^d(j_e) \bar{u}_i(j_e) \bar{q}_i(j_e, \bar{u}_i(j_e)) \Pi_e(j_e). \end{aligned} \tag{6.2}$$

A6.3. Example 2.2, including partially known channel state and bit interval control. *There is an admissible non-state-dependent power and time allocation* $\{\bar{u}_i(j_e), \bar{v}_i(j_e), i \leq K\}$, *all* j_e, *such that* $\sum_i \bar{u}_i(j_e) = \bar{u}$, $\sum_i \bar{v}_i(j_e) = 1$, *and for all* i

$$\bar{\lambda}_i^a < \sum_{j_e} \bar{\lambda}_i^d(j_e)\bar{v}_i(j_e)\bar{u}_i(j_e)\bar{q}_i(j_e, \bar{u}_i(j_e))\Pi_e(j_e). \tag{6.3}$$

6.1 Example 2.1: Known Channel State

Example 6.1. Consider Example 2.1, and assume (A6.1) with known channel state. For some $p > 1$ (which need not be an integer), define $V(\bar{x}) = \sum_i a_i(\bar{x}_i)^{p+1}$, $a_i > 0$. Then at the scheduling times $\mu\bar{\Delta}^{s,n}$ with queue state $\bar{x}$ the derivative $V'_{\bar{x}}(\bar{x})b(\bar{x}, u(\bar{x}))$ equals

$$(p+1)\sum_i a_i\,(\bar{x}_i)^p \left[\bar{\lambda}_i^a - \sum_j \bar{\lambda}_i^d(j)u_i(j,\bar{x})q_i(j,u_i(j,\bar{x}))\Pi(j)\right]. \tag{6.4}$$

The perturbed Liapunov function (5.5) has the form

$$V^n(k\delta_n) = \sum_i a_i\,[\bar{x}_i^n(k\delta_n)]^{p+1} + \sum_i [\bar{x}_i^n(k\delta_n)]^p\, c_i^n(k\delta_n),$$

where the $c_i^n(\cdot)$ are bounded, uniformly in k, n, ω. The Liapunov function method seeks a control that makes (6.4) as negative as possible for large $\bar{x}$. Thus, for each j maximize

$$\sum_i a_i\bar{\lambda}_i^d(j)u_i(j,\bar{x})q_i(j,u_i(j,\bar{x})) \tag{6.5}$$

over the admissible controls. If $q_i(j,u_i(j,\bar{x})) \equiv 1$, then the maximization is equivalent to applying the maximum possible (i.e., subject to the constraints) power to

$$\arg\max_i \left\{a_i(\bar{x}_i)^p\bar{\lambda}_i^d(j)\right\}. \tag{6.6}$$

If there is power left over, then apply it to the next best, etc. The control given by (6.6) will be discontinuous. If $q_i(j,u_i(j,\bar{x})) \equiv 1$ and there are no constraints, then for each channel state j the sets of constant control value will be convex with piecewise planar boundaries. The theorems of Section 5 require smooth controls. But, just smooth it in a small neighborhood of the sets of discontinuities. Alternatively, see the remarks in Subsection 6.3 which show how to extend the proof to allow discontinuous controls. Under the conditions on the input and channel processes, all of the other conditions of Theorem 5.1 hold. Hence, (A6.1) is sufficient for stability.

If there are constraints on the power allocation or if $q_i(j,\cdot)$ is not identically unity, then the problem is more complicated. We need to maximize (6.5) over the possibilities. But (A6.1) is still sufficient for stability.

Note on optimization. The Liapunov function can be considered to be the (total or relative) cost function for an optimal control problem [7]. In the present case, due to the error terms in (5.16), the cost criteria associated with $V(\cdot)$ cannot be precisely identified. But for the $V(\cdot)$ of this example the dominant component is the negative of (6.4). Let $q_i(j, u_i(j, \bar{x})) \equiv 1$. Then, the larger p is, the more weight is given to the largest queue, for all channel states. One can select the value of p to balance fairness (essentially equal allocations to the queues) with a desire to avoid some queues being much longer than others.

Example 6.1, a two-dimensional case. Let $a_i = i$, all i. Consider a two-queue and two-channel state example and let $q_i(j, u_i(j, \bar{x})) \equiv 1$, with no constraints on the power allocation. Let $\bar{\lambda}_1^d(1) > \bar{\lambda}_2^d(1)$, and $\bar{\lambda}_1^d(2) < \bar{\lambda}_2^d(2)$. For channel state 1, apply all power to queue 1 when

$$\frac{\bar{x}_1}{\bar{x}_2} \geq \left[\frac{\bar{\lambda}_2^d(1)}{\bar{\lambda}_1^d(1)}\right]^{1/p}. \tag{6.7}$$

For channel state 2, apply all power to queue 2 when

$$\frac{\bar{x}_2}{\bar{x}_1} \geq \left[\frac{\bar{\lambda}_1^d(2)}{\bar{\lambda}_2^d(2)}\right]^{1/p}. \tag{6.8}$$

As $p \to \infty$, the switching lines move to the diagonal.

An alternative Liapunov function. Now consider the Liapunov function $V(x) = \sum_i [\bar{x}_i + c_i]^{p+1}$. With equal c_i, this gives results that are closer to the numerically computed optimal controls for cost rate $\min\{\bar{x}_1, \bar{x}_2\}$. For the two-queue- and two-channel-state example above, The inequalities (6.6) and (6.7) are replaced by

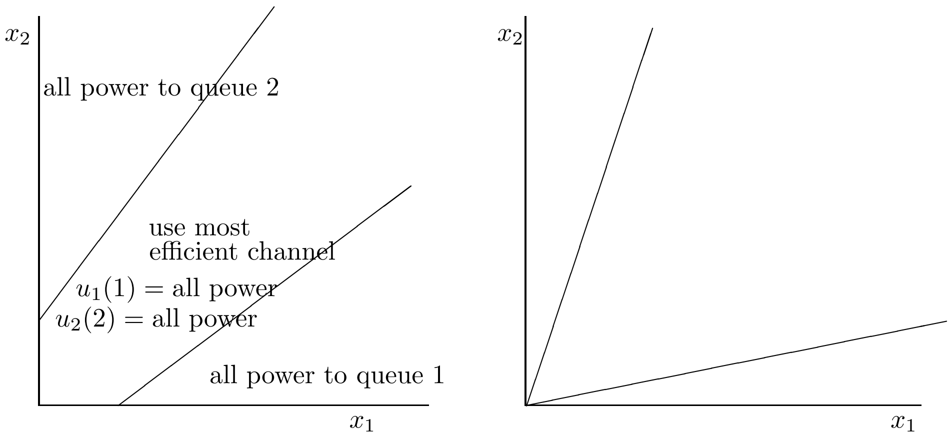
$$\frac{\bar{x}_1 + c_1}{\bar{x}_2 + c_2} \geq \left[\frac{\bar{\lambda}_2^d(1)}{\bar{\lambda}_1^d(1)}\right]^{1/p}, \tag{6.9}$$

$$\frac{\bar{x}_2 + c_2}{\bar{x}_1 + c_1} \geq \left[\frac{\bar{\lambda}_1^d(2)}{\bar{\lambda}_2^d(2)}\right]^{1/p}. \tag{6.10}$$

Here the cost rate is less sensitive to differences in the $\bar{x}_i$ unless they are large.

Comparison with optimal controls. The following figure gives the optimal switching surfaces for the same problem with $\bar{\lambda}_1^d(1) > \bar{\lambda}_2^d(1), \bar{\lambda}_2^d(2) > \bar{\lambda}_1^d(2)$, obtained by numerical solution of the limit control problem using the methods of [9]. The system is (3.8) with white noise of various intensities added. The qualitative form of the controls did not depend on the intensity of the noise. The cost rate is $k(x) = \max\{x_1, x_2\}$ for the left-hand figure and $\sum_i x_i$ for the

right-hand figure. The total cost was either of the ergodic or discounted form, with little difference between them in the form of the controls. The slopes for the linear cost rate tend to be larger than those for the $\max\{x_1, x_2\}$ criterion. Equivalently, for the linear cost rate we put more emphasis on using the most efficient channel and less on attaining closeness of the queues. Mixtures of the criteria yield intermediate switching curves. Note the similarity of the controls to the forms given by ((6.7), (6.8)) and ((6.9), (6.10)).



Switching curves for the optimal control:
$k(x) = \max\{x_1, x_2\}$ and $\sum_i x_i$.

Partially unknown channel state. Assume (A6.2) in lieu of (A6.1) and refer to the comments at the end of Subsection 5.1. The method is precisely the same, except that the control is based on the estimate j_e and not on the unknown true channel state j.

6.2 Example 2.2

Use $V(\bar{x}) = \sum_i a_i(\bar{x}_i)^{p+1}$. Then (6.4) is replaced by

$$(p+1)\sum_i a_i(\bar{x}_i)^p \left[\bar{\lambda}_i^a - \sum_j \bar{\lambda}_i^d(j_e)u_i(j_e,\bar{x})v_i(j_e,\bar{x})\bar{q}_i(j_e,u_i(j_e,\bar{x}))\right] \tag{6.11}$$

and (6.5) by

$$\sum_i a_i\bar{\lambda}_i^d(j_e)u_i(j_e,\bar{x})v_i(j_e,\bar{x})\bar{q}_i(j_e,u_i(j_e,\bar{x})) \tag{6.12}$$

The method is exactly the same as for Example 2.1. For example, suppose that $\bar{q}_i(j_e, u_i(j_e,\bar{x})) \equiv 1$. Then, in the absence of constraints apply as much *power and time* as possible to

$$\arg\max_i \left\{a_i(\bar{x}_i)^p\bar{\lambda}_i^d(j_e)\right\}. \tag{6.13}$$

6.3 Discontinuous Controls

Under additional conditions, the smoothness condition on $u_i(j,\cdot)$ in (A4.3) can be dropped. This will be illustrated by the case of Example 6.1, where there are no constraints and $q_i(j,u) = 1$ for all i, j, u. The smoothness was used only in dealing with (5.12). Let us use the decision rule (6.6). Let $i_j(\bar{x})$ denote the value of i for which $u_i(j,\bar{x}) = \bar{u}$ when the channel state is j and the queue state is $\bar{x}$. By the other conditions of Theorem 5.1, the contribution of (5.12) to the error term in (5.16), summed over i, j, can be approximated by

$$\begin{aligned}
&\bar{u}\sum_j E^n_{\mu\bar{\Delta}^{s,n}} V_{\bar{x}_{i_j(\bar{x}^n(\mu\bar{\Delta}^{s,n}+\bar{\Delta}^{s,n}))}}(\bar{x}^n(\mu\bar{\Delta}^{s,n}+\bar{\Delta}^{s,n})) \\
&\qquad\times\bar{\lambda}^d_{i_j(\bar{x}^n(\mu\bar{\Delta}^{s,n}+\bar{\Delta}^{s,n}))}(j) C^{d,0,n}_{\mu\bar{\Delta}^{s,n}+\bar{\Delta}^{s,n}}(j) \\
&-\bar{u}\sum_j E^n_{\mu\bar{\Delta}^{s,n}} V_{\bar{x}_{i_j(\bar{x}^n(\mu\bar{\Delta}^{s,n}))}}(\bar{x}^n(\mu\bar{\Delta}^{s,n}))\bar{\lambda}^d_{i_j(\bar{x}^n(\mu\bar{\Delta}^{s,n}))}(j) C^{d,0,n}_{\mu\bar{\Delta}^{s,n}+\bar{\Delta}^{s,n}}(j),
\end{aligned} \tag{6.14}$$

plus an error which is strictly dominated by 1/8 times the right side of (5.17) for large $\bar{x}^n(\mu\bar{\Delta}^{s,n})$ and n. By the conditions of Theorem 5.1, the (sum over j of the) difference between the jth summands of the two terms in (6.14) is also strictly dominated by 1/8 times the right side of (5.17) for large $\bar{x}^n(\mu\bar{\Delta}^{s,n})$ and n. Then the form (5.17) can still be obtained for large $\bar{x}^n(\mu\bar{\Delta}^{s,n})$ and n.

Further development is beyond our scope here.

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